

## Thermal analysis of Stirling engine solar driven

Mohamed Abbas<sup>1\*</sup>, Noureddine Said<sup>2</sup> and Boussad Boumeddane<sup>3</sup>

<sup>1</sup> Unité de Développement des Equipements Solaires, 'UDES'  
Route Nationale N°11, B.P. 365, Bou Ismail, Tipaza, Algérie

<sup>2</sup> Division Thermique et Thermodynamique, Centre de Développement des Energies Renouvelables, 'CDER'  
B.P. 62, Route de l'Observatoire, Bouzaréah, Alger, Algérie

<sup>3</sup> Département de Mécanique, Université Saâd Dahlab, Route de Soumâa, Blida, Algérie

(reçu le 30 Novembre 2007 – accepté le 25 Décembre 2008)

**Abstract** - *Solar energy is one of the more attractive renewable energy sources; the conversion of the latter per thermal way into electricity is a major energy stake. The current systems are primarily based on technology known as 'solar dish/Stirling', which uses Stirling engines placed at the focal plan of a parabolic concentrator. The Stirling engine presents an excellent theoretical output equivalent to the output of Carnot one. It is with external combustion, less pollutant, silencer and request little maintenance. Thanks to these advantages which the Stirling engine is very interesting to study. The dish Stirling system studies consist on three parts; the thermal modelling of Stirling engine, optical study of parabolic concentrator and finally the thermal study of the receiver. The present study is dedicated only to a thermal modelling of the Stirling engine based on the decoupled method. We evaluate, starting from an ideal adiabatic analysis, the thermal and mechanical powers exchanged, that we correct then by calculating the various losses within the machine. This model led to the writing of important set of equations algebra - differentials. The calculation programme worked out under Fortran to solve this system, makes allow to calculate the performances of any types of the Stirling engines, according to the kinematics used, the types of regenerators, the exchangers, as well as the various working liquids used.*

**Résumé** - *L'énergie solaire est l'une des plus attrayantes des sources d'énergie renouvelables. La conversion de cette dernière par voie thermique en énergie électrique est l'un des principaux enjeux. Les systèmes actuels sont principalement basés sur la technologie connue sous le nom de 'Solar Dish / Stirling', qui utilise les moteurs Stirling placés au plan focal d'un concentrateur parabolique. Le moteur Stirling présente une excellente production théorique équivalente à la sortie de Carnot. Il est à combustion externe, moins polluant, silencieux et demande peu d'entretien. Tenant compte de ces avantages, le moteur Stirling est intéressant à étudier. L'étude du système Stirling se compose de trois parties: la modélisation thermique du moteur Stirling, l'étude optique du concentrateur parabolique et enfin l'étude thermique du récepteur. La présente étude est consacrée uniquement à une modélisation thermique du moteur Stirling, basée sur la méthode découplée. Nous évaluons, à partir d'une analyse adiabatique idéale, les contraintes thermiques et mécaniques des puissances échangées, alors que nous corrigeons par le calcul les différentes pertes au sein de la machine. Ce modèle a conduit à la rédaction d'importants systèmes d'équations différentielles. Le programme de calcul élaboré en Fortran pour la résolution de ce système, permet d'évaluer et de calculer les performances de tous les types de moteurs Stirling, en fonction de la cinématique utilisée, les types de régénérateurs, d'échangeurs, ainsi que les divers fluides liquides de travail utilisés.*

**Keywords:** Moteur Stirling – Solar Dish – Concentrateur parabolique.

---

\* [abbasdreams2003@yahoo.ca](mailto:abbasdreams2003@yahoo.ca)

## 1. INTRODUCTION

The rapid depletion of our natural resources has focused attention on new energy sources and on effective means of energy conservation. One of the means of effective energy conservation is the Stirling engine [1].

A Stirling cycle machine operates on a closed regenerative thermodynamic cycle using a working gas, and subjects the gas to expansion and compression processes at different temperatures. Since Stirling engines are externally heated, environmentally very clean engine having high theoretical cycle efficiency, they can be powered using a wide variety of fuels and heat sources such as, combustible materials, solar radiation, geothermal hot water, radioisotope energy [2]. The Stirling heat engine was first patented in 1816 by Robert Stirling. Since then, several Stirling engines based on his invention have been built in many forms and sizes [3]. In solar modules, Stirling-Dish, the solar radiation is converted to electricity in three stages.

In the first stage, radiation is converted to heat by focusing the solar radiation onto a light absorbing heat pipe by means of a parabolic reflector. In the second stage, the heat is converted to mechanical power by a Stirling engine. In the final stage, the mechanical power is converted to electricity by an alternator.

The techniques of analysis for Stirling engines can be categorized with Martini's [4] nomenclature as follows:

- Zero the Order Analysis: as Beale formula.
- First Order Analysis: (Schmidt analysis) was done in 1871 by Gustav Schmidt in which he obtained closed-form solutions for the special case of sinusoidal volume variations and isothermal hot and cold spaces.
- Second Order Analysis (decoupled methods): This level of analysis may be based on an adiabatic analysis that subtracts losses caused by heat transfer and flow power losses. It relies on a modified Schmidt analysis and requires nonlinear time integration of the model equations. It assumes adiabatic expansion and compression regions.
- Third order analysis (coupled methods): uses control volumes or nodes to directly solve one-dimensional (2D and 3D) governing equations. At this level of modelling, the use of the computer codes is very necessary such as GLIMPS, HFAST, CAST, FLUENT, STAR, CFX.

In the present study the thermodynamic modelling of the Gamma type of Stirling-cycle engine based on the decoupled method is performed. We evaluate, starting from an ideal adiabatic analysis, the thermal and mechanical powers exchanged, that we correct then by calculating the various losses within the machine [5].

The engine is divided into 5 control volumes, in addition the regenerator is divided into three subdivisions, as shown in Fig. 1 [6]. The engine consists respectively of an expansion space, E; heater H; regenerator, R; cooler, K; and a compression space, C. Heat is transferred from the external heat source to the working fluid in the heater section, cyclically stored and recovered in the regenerator, and rejected by the working fluid to the external heat sink in the cooler section.

The control volumes of compression and expansion spaces are variable volumes, and the volumes of H and K have fixed volumes [7].

The conservation of mass and energy equation are written for each control volume of the Engine [5, 6]. A computer program is written in Fortran, and the equations are solved iteratively.

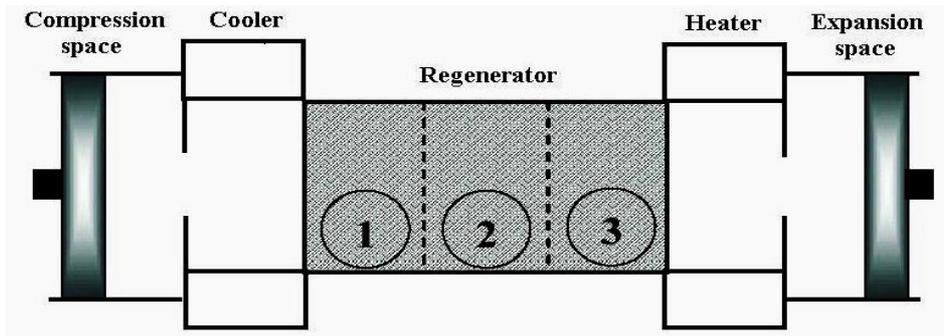


Fig. 1: Stirling engine configuration

## 2. THEORY

### 2.1 Ideal adiabatic model

The adiabatic model relies on the following usual main assumptions [5]:

- The processes are considered to be reversible.
- The ideal gas law can be applied to the working fluid.
- The instantaneous pressure is uniform in the engine.
- The compression and expansion spaces are adiabatic.

The general approach for deriving the equation set is to apply the equations of energy and state to each of the control volume. The resulting equations are linked by applying the continuity equation across the entire system [6].

In this ideal model, the fluid temperature in the heat exchangers are taken as uniform and constant, in the regenerator a constant linear fluid temperature profile is assumed between  $T_H$  and  $T_K$ .

The mean effective fluid temperature in the three parts of regenerator is thus obtained as the logarithmic mean temperature between  $T_H$ ,  $T_1$ ,  $T_2$  and  $T_K$ , for example:

$$T_{R1} = \frac{T_1 - T_K}{\ln(T_1/T_K)} \quad (1)$$

The variable volume expansion and compression spaces; C and E depend on the kinematics used, in the case of Swash plate, it's expressed as respectively:

$$V_E = V_{DEAE} + \frac{V_{SWE}}{2} \times (1 + \cos(\theta + \alpha)) \quad (2)$$

$$V_C = V_{DEAC} + \frac{V_{SWC}}{2} \times (1 + \cos(\theta)) \quad (3)$$

$$\text{The equation of state is: } P V = m R T \quad (4)$$

The total mass of gas is constant and equal to:

$$M = m_C + m_K + m_{R1} + m_{R2} + m_{R3} + m_H + m_E \quad (5)$$

The instantaneous pressure of the engine is:

$$P = M \times R \times \left( \frac{V_C}{T_C} + \frac{V_K}{T_K} + \frac{V_R}{T_R} + \frac{V_H}{T_H} + \frac{V_E}{T_E} \right)^{-1} \quad (6)$$

The equations of the mass flow rate of the interfaces are obtained by the method suggested by W.R. Martini [5]. They are given for  $i$  control volume by:

$$\dot{m}_{i+1} = \dot{m}_i - \frac{d(m_{i,i+1})}{dt} \quad (7)$$

$$\text{For example: } \dot{m}_{CK} = -d(m_C) \quad (8)$$

The energy equation applied to a generalised control volume is written as:

$$dQ + (C_p \cdot \dot{m}_{in} \cdot T_{in} - C_p \cdot \dot{m}_{out} \cdot T_{out}) = dW + d(m \cdot C_v \cdot T) \quad (9)$$

This equation is written for each control volume to calculate the amount of heat transferred in each element of Stirling engine, for example:

$$dQ_K = V_k \cdot C_v \cdot \frac{dp}{R} - C_p \cdot (T_{CK} \cdot \dot{m}_{CK} - T_K \cdot \dot{m}_{KR}) \quad (10)$$

The work done in the compression and expansion cells is:

$$\delta W_E = -P \times \left( \frac{W_{SWE}}{2} \times \sin(\theta + \alpha) \right) \quad (11)$$

$$\delta W_C = -P \times \left( \frac{W_{SWC}}{2} \times \sin(\theta) \right) \quad (12)$$

We define the indicated efficiency as follows:

$$\eta = \frac{W}{Q_H} \quad (13)$$

## 2.2 Decoupled analysis

In this section we adopt a 'Quasi-Steady Flow' approach, in that we assume that at each instant of the cycle the fluid behaves as though it is in steady flow.

### 2.2.1 Non - ideal heater and cooler

We consider that the fluid temperature is different then the wall temperature in both exchangers. We evaluate the new values of gas temperatures  $T_K$  and  $T_H$  as follow:

$$T_K = T_{WK} - \frac{Q_K}{h_K \cdot A_{wgk}} \quad (14)$$

$$T_H = T_{WH} - \frac{Q_H}{h_H \cdot A_{wgh}} \quad (15)$$

### 2.2.2 Non - ideal regenerator

The relationship of regenerator effectiveness  $\varepsilon$  and thermal efficiency  $\eta$  is:

$$\eta = \frac{\eta_i}{\left[ 1 + \left( \frac{Q_{Ri}}{Q_{Hi}} \right) \times (1 - \varepsilon) \right]} \quad (16)$$

We can assimilate the regenerator as a counter-flow exchanger, its effectiveness is:

$$\varepsilon = \frac{NUT}{1 + NUT} \quad (17)$$

$$\text{With: } NUT = \frac{N_{ST} \times \left( \frac{A_{WR}}{A_R} \right)}{2} \quad (18)$$

### 2.2.3 Actual thermal power

The actual thermal power is the sum of basic thermal power and different thermal losses:

$$\dot{Q}_{act} = \dot{Q}_i + \sum \dot{Q}_R + \sum \dot{Q}_{COND} + \dot{Q}_{PUMP} + \dot{Q}_{SHUT} \quad (19)$$

#### a. Shuttle heat transfer loss

It's given by:

$$\dot{Q}_{SHUT} = \frac{0.4 \times S_L^2 \times k_g \times d_p \times (T_E - T_C)}{x_g \times L_p} \quad (20)$$

#### b. Regenerator net enthalpy loss

Can be expressed by:

$$\dot{Q}_R = \dot{m}_R \times C_p \times (T_{WH} - T_{WK}) \times (1 - \varepsilon) \quad (21)$$

#### c. Regenerator wall heat leakage

Can be modelled by:

$$\dot{Q}_{COND} = k_j \times A_j \times \left( \frac{T_{chaud} - T_{froid}}{L_j} \right) \quad (22)$$

#### d. Pumping losses

Can be expressed by Leo formula [5]:

$$\dot{Q}_{\text{PUMP}} = \left( \frac{\pi \cdot d_p}{k_g} \right)^{0.6} \times \frac{2L_p \cdot (T_E - T_C)}{1.5} \times \left( \frac{2(P_{\text{max}} - P_{\text{min}}) \cdot \text{freq} \cdot C_p}{R(T_E - T_C)} \right)^{1.6} \times x_g^{2.6} \quad (23)$$

### 2.2.4 Actual mechanical power

The actual mechanical power is equal to:

$$\dot{W}_{\text{Act}} = \dot{W}_i - \sum \dot{W}_w - \dot{W}_{\text{fric}} \quad (24)$$

#### a. Pressure drop loss

We can determine the new value of work done by integrating over the complete cycle, and isolate the pumping loss term as follows :

$$W = \oint P \times (d(V_C) + d(V_E)) - \oint \sum \Delta P \times d(V_E) = W_i - \Delta W \quad (25)$$

$$\text{where: } \Delta W = \int_0^{2\pi} \left( \sum_{i=1}^5 \Delta P_i \times \frac{d(V_E)}{d\theta} \right) \times d\theta \quad (26)$$

The pressure drop in the 3 exchangers can be expressed as:

$$\Delta P = \frac{2C_{\text{ref}} \times \mu \times u \times V}{d_h^2 \times A} \quad (27)$$

$C_{\text{ref}}$  is define according to the following equation:

$$C_{\text{ref}} = \text{Re} \times C_f \quad (28)$$

## 3. NUMERICAL METHOD

We must solve a system of 30 non linear equations among which 10 differential equations that should be integrated numerically for specific configurations and operating conditions.

Because of its cyclic nature, the system can be formed as an initial value problem by assigning arbitrary initial conditions, and integrating the equations through several complete cycles until a cyclic steady state has been attained.

According to Ureili et Berchowitz [5], the most sensitive measure of convergence to cyclic steady state is the residual regenerator heat  $Q_R$  at the end of the cycle, which should be set to zero.

## 4. RESULTS AND DISCUSSIONS

The computer program prepared in Fortran is applied to a Stirling engine known as 'Organ machine' [8]. Air is used as the working gas, the compression and expansion space initial gas temperatures are taken as 875 and 285 K, respectively. The results are obtained using the parameters given in **Table 1**.

The simulation of a kinematics Stirling engine led to the P-V diagram given in Fig. 2. We can notice however that the diagram obtained differs already appreciably

from that of the theoretical cycle of Stirling, although does not correspond yet to the real diagram.

**Table 1:** Data used to analysis the Stirling engine [6, 11]

Type engine	Gamma
Work fluid	Hydrogen
Engine speed	2000 rpm
Average pressure	15 MPa
Phase angle	90 degrees
Swept volume per cylinder	134 CC
Compression dead volume	28 CC
Expansion dead volume	33 CC

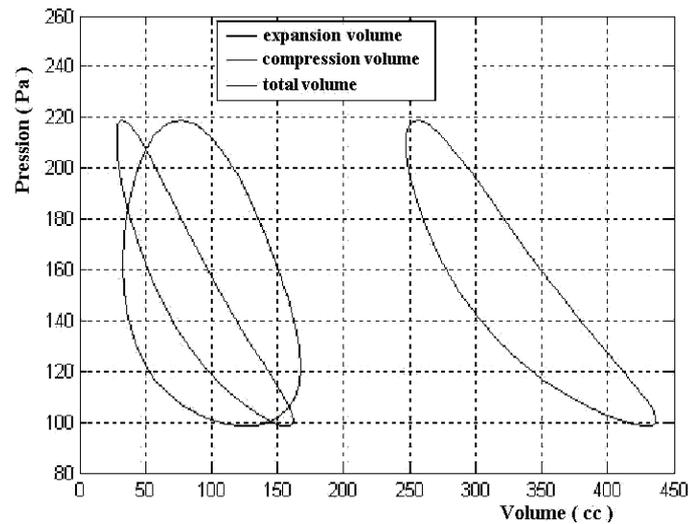


Fig. 2: P–V diagram

The variation of temperatures of the control volumes with crank angle are shown in Fig. 3. We can remark that:

- The expansion control volume temperature is on average less low than that of the wall of the heater but there is however part of the cycle where it is higher.
- The cyclic variation of the gas temperature in the expansion space is larger than space compression one.

Fig. 4 show the mass flow rate variation in different spaces with crank angle, the figure show that the variation amplitude of flow rates is larger in the cold space. The positive flow rate period is more important than the negative flow rate one; and there are values of the phase for which all the flow rates are equal. This phenomenon occurs at the time of each extremum of pressure.

Fig. 5 shows the heats accumulate transferred over the cycle. The most significant aspect of this diagram is the:

- Considerable amount of heat transferred in the regenerator over the cycle. This tends to indicate that the engine performance depends critically on the regenerator effectiveness.
- Significantly the energy rejected by the gas to the regenerator matrix in the first half of the cycle is equal to the energy absorbed by the gas from the matrix in the second half of the cycle, thus the net heat transfer to the regenerator over a cycle is zero.

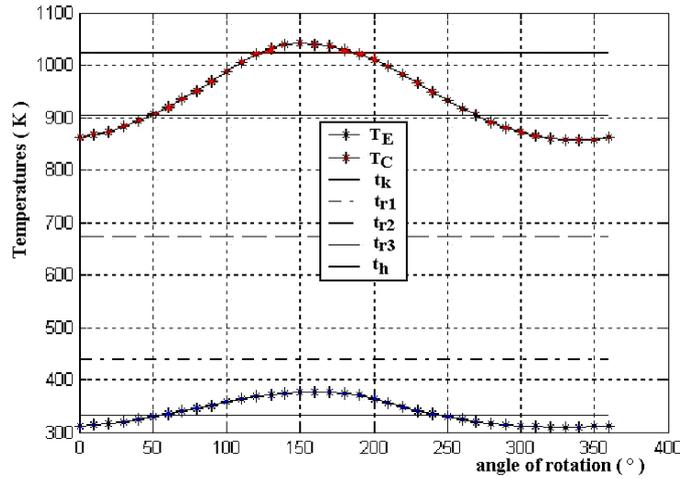


Fig. 3: Temperatures variation in Stirling engine

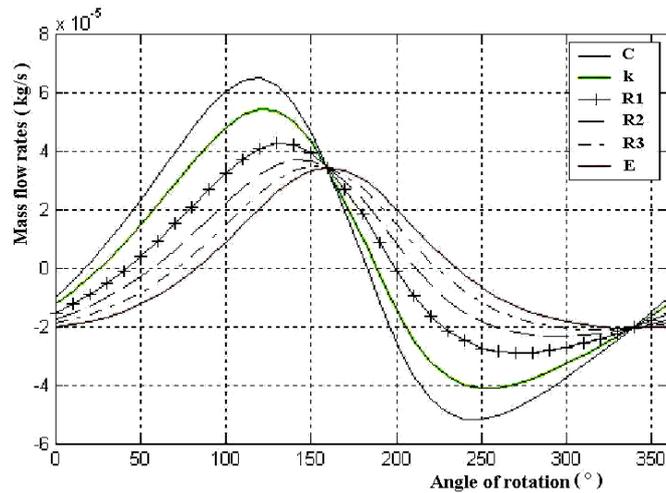


Fig. 4: Flow rates in controls volume of engine

Influence of losses on the Stirling engine performances

Fig. 6 illustrates the influence of the imperfection of cooler and heater on the Stirling engine performances. Notice that the mean temperature of the gas in the heater space is 53.9 degrees below that of the heater wall, and similarly the mean temperature of the gas in the cooler space is 11 degrees above that of the cooler wall. This lower temperature range of operation reduced the output power, and the thermal efficiency.

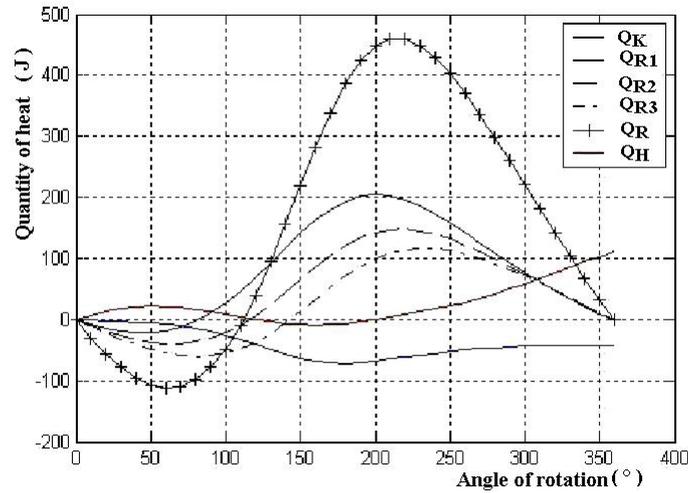


Fig. 5: Heat accumulated in engine

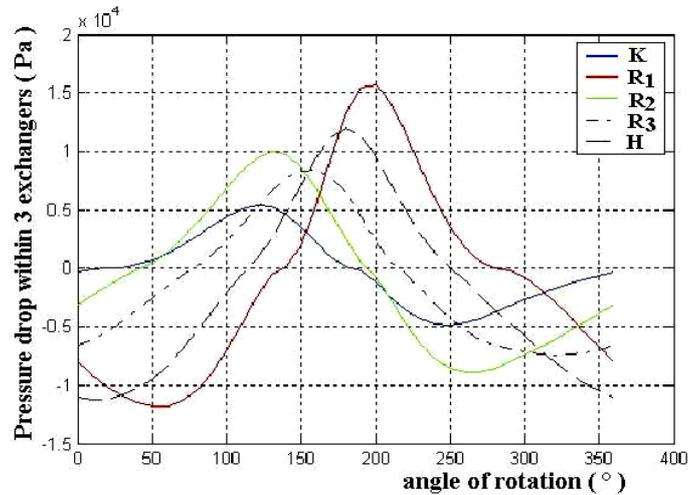


Fig. 6: Influence of the imperfection of heater and cooler

Fig. 7 illustrates the pressure drop in the 3 heat exchangers in Stirling engine. Note the relative magnitude of the regenerator pressure drop with respect to those of the heater and cooler.

Fig. 8 shows the effect of regenerator effectiveness  $\epsilon$  on thermal efficiency  $\eta$ . Notice that the thermal efficiency  $\eta$  drops from more than 60 % to less than 10 %. Thus, we see that for highly effective regenerators a 1 % reduction in regenerator effectiveness results in a more than 5 % reduction in thermal efficiency  $\eta$ .

Furthermore we see that if one has a regenerator with an effectiveness of 0.8, the thermal efficiency will drop by half to around 30 %. This means a significantly less efficient machine. Obviously we need to have means of determining the actual regenerator effectiveness in any specific machine [7].

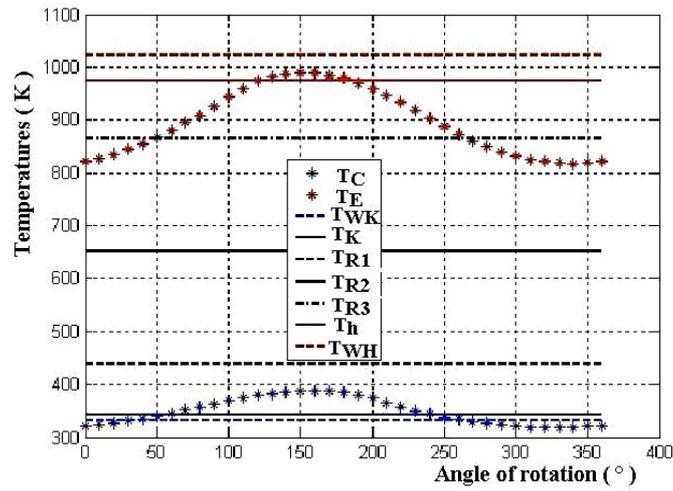


Fig. 7: Pressure drop in heat exchangers

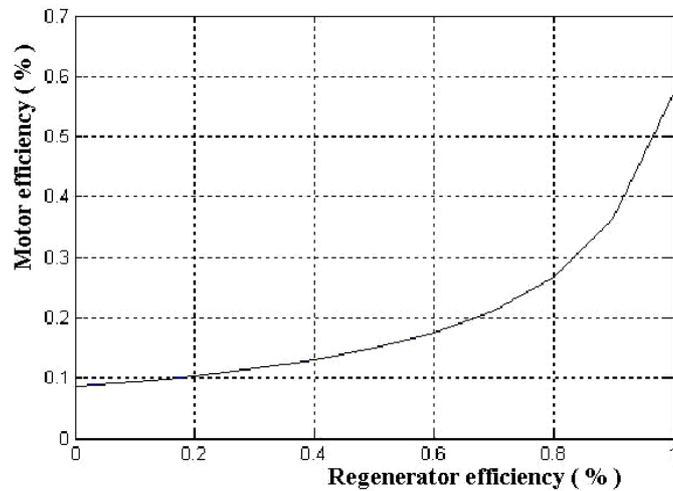


Fig. 8: Regenerator effectiveness influence

## 5. CONCLUSION

The following conclusions can be derived from the analysis:

- The analysis provides the necessary data for the comparison of several aspects of the Stirling-cycle engine.
- The Stirling engine can be designed using the results of this analysis.
- The regenerator effectiveness must be more than 80%.
- The progression of the gas work pressure must be ensured by a good sealing.
- The phase angle must be 90 degrees.
- A sensitivity study of different parameters is necessary to know their influence to the Stirling engine performances.

**Acknowledgement:** The support of UDES (Unité de Développement des Equipements Solaires), CDER (Mr N. Said) and Blida University (Mr Boumedanne) is gratefully acknowledged.

## NOMENCLATURE

A	Area (section), m <sup>2</sup>	i	Ideal, number of control volume
C <sub>f</sub>	Friction coefficient	K	Cooler
C <sub>ref</sub>	Reynolds friction coefficient	H	Heater
C <sub>p</sub>	Pressure constant heat capacity, J.kg <sup>-1</sup> .K <sup>-1</sup>	HE	Interface between heater and expansion space
C <sub>v</sub>	Volume constant heat capacity, J.kg <sup>-1</sup> .K <sup>-1</sup>	KR	Interface between part 1 of regenerator and cooler
m	Masse, kg	Pump	Pumping
$\dot{m}$	Flow rate, kg.s <sup>-1</sup>	R	Regenerator
NUT	Number unit transfer	R1	Part 1 of regenerator
N <sub>st</sub>	Stanton Number	R2	Part 2 of regenerator
P	Pressure, Pa	R3	Part 3 of regenerator
Q	Heat, J	R1R2	Interface between part 1 and part 2 of regenerator
R	Ideal gas constant, J.kg <sup>-1</sup> .K <sup>-1</sup>	R2R3	Interface between part 1 and part 3 of regenerator
Re	Reynolds number	R3H	Interface between part 3 of regenerator and heater
S <sub>L</sub>	Piston course, m	sw	Swept volume
W	Work, J	Shut	Shuttle losses
x <sub>g</sub>	Displacers-cylinder clearance, m	wg	Wetted area
	<i>Subscripts and Superscripts</i>	Wk	Cooler wall
Act	Actual	Wh	Heater wall
C	Compression space		
CK	Interface between compression space and cooler		
dea	Dead volume		
E	Expansion space	$\alpha$	Phase angle piston-displacer, rad
F	Friction	$\varepsilon$	Regenerator effectiveness
g	Gas	$\eta$	Engine thermal efficiency
		$\theta$	Rotation angle, m

### *Greek symbols*

## REFERENCES

- [1] C. Çinar and H. Karabulut, 'Manufacturing and Testing of a Gamma Type Stirling Engine', Renewable Energy, Vol. 30, N°1, pp. 57 – 66, 2005.
- [2] Ö.E. Ataer and H. Karabulut, 'Thermodynamic Analysis of the V-Type Stirling-Cycle Refrigerator', International Journal of Refrigeration, Vol. 28, N°2, pp. 183 – 189, 2005.
- [3] B. Kongtragool and S. Wongwises, 'A Review of Solar-Powered Stirling Engines and Low Temperature Differential Stirling Engines', Renewable and Sustainable Energy Reviews, Vol. 7, N°2, pp. 131 – 154, 2003.
- [4] R.W. Dyson, S.D. Wilson and C.T. Roy, 'Review of Computational Stirling Analysis Methods', NASA/TM—2004-213300, AIAA—2004—5582.

- [5] M. Abbas, '*Simulation Numérique d'un Cycle de Moteur Stirling Solaire de Type Gamma par la Méthode Découplée*', Mémoire de Magister, Université de Blida, 2006.
- [6] I. Ureili and D.M. Berchowitz, '*Stirling Cycle Analysis*', Published by Adam Hilger Ltd. Bristol. Version actualisée de ce livre est disponible sur le site personnel du professeur Israël Ureili: [www.ent.ohiou.edu](http://www.ent.ohiou.edu)
- [7] S. Bonnet, '*Moteurs Thermiques à Apport de Chaleur Externe: Etude d'un Moteur Stirling et d'un Moteur Ericsson*', Université de Pau et des Pays de l'Adour, 2005.
- [8] P. Stouffs, '*Conception et Réalisation d'un Moteur Stirling de 1 kW Electrique Destiné à la Production Combinée d'Electricité et d'Eau Chaude à Partir d'Energie Solaire*', Journées d'Etudes SFT, France, 14 Octobre 1999.