



Heat Transfer Enhancement for Rarefied Flow Within a Microchannel Featuring Obstructions

Siham Hammid^{a,*}, Khatir Naima^b, Abdelkrim Liazid^c, Cheikh Kezrane^a

^a *Laboratory of Development in Mechanics and Materials (LDMM), Zian Achour University, Djelfa PB 3117, Algeria*

^b *Department of Technology, University Center Salhi Ahmed Naama (Ctr. Univ. Naama), P.O. Box 66, Naama, 45000, Algeria*

^c *Departement of Physics, Faculty of Sciences Abou Bekr Belkaid University, 22 Rue Abi Ayed Abdelkrim, 13000, Tlemcen, Algeria*

ARTICLE INFO

Article history:

Received September 29, 2024

Accepted October 16, 2024

Keywords:

Microchannel,
LBM,
Laminar flow,
Obstacle,
Knudsen number.

ABSTRACT

The present study investigates forced convection heat transfer of rarefied flow within a microchannel containing obstacles using the thermal lattice Boltzmann method using a double distribution function model and BGK approximation. Slip velocity and temperature jump conditions were employed across microchannel walls. The microchannel temperature and velocity input are constant. The microchannel configuration has three obstacles imposed along the lower microchannel wall. The study simulates rarefied fluid flow and heat transmission of forced convection inside the microchannel, considering separation between obstacles as the primary study objective. The findings represent the distribution of temperature and velocity. In addition, temperature jump and slip velocity in the function of Knudsen numbers were also represented. The findings highlight the substantial influence of barriers on temperature and velocity. As the distance between obstacles drops, the temperature diminishes. Additionally, a rise in separation distances significantly aids in the dropping of velocity. The results reveal a significant reduction in slip velocity as Knudsen numbers increase across the microchannel length. The outcomes of the present investigation could assist and be used as a cooling solution for various technologies, such as microelectronics and nanoelectromechanical systems. Additionally, the suggested configuration might be utilized to improve microfluidic device design.

* Corresponding author, E-mail address: siham.hammid@univ-djelfa.dz

Tel : + 213 674246319



1. INTRODUCTION

Fluid flow comprehension and heat transfer in microgeometry have become critical due to the growing popularity of miniaturized electronics (Ghadirzadeh & Kalteh, 2017). Researchers have recently investigated the fluid flow and heat transmission characteristics in microchannels due to their multiple uses across bioengineering, microelectronic cooling techniques, and also micro-heat exchangers[6](Hammid, Naima, Alqahtani, et al., 2024). At the microscale, fluid flow and heat transfer behaviour exhibit substantial differences compared to the traditional macroscale, leading to invalidating the continuity hypothesis (Hammid, Naima, Alqahtani, et al., 2024). The prominence of the effect of rarefaction effect is more significant at this level and is represented by the Knudsen number(kn), which is calculated as the ratio of the mean free path to the characteristic length(Hammid, Naima, Ikumapayi, et al., 2024) (Ghadirzadeh & Kalteh, 2017; Sohankar, Joulaei, & Mahmoodi, 2022).

At the micro-scale, the Knudsen number Kn serves as an indicator for fluid flow classification. The categories are continuum flow, which applies to Kn numbers below 0.001; slip flow, which applies to Kn numbers ranging from 0.001 return to 0.1; and Kn numbers ranging from 0.01 to 10 for transition flow; and free- molecular flow, which applies to Kn numbers above 10.(Hammid, Naima, Ikumapayi, et al., 2024)

The Lattice Boltzmann method (LBM) has become a robust computational approach for simulating heat transfer and engineering challenges(Abaszadeh, Safavinejad, Amiri, & Amiri Delouei, 2022). LBM offers several attractive features, such as a more accurate representation of small interactions (Afra, Delouei, & Tarokh, 2022; Afra, Karimnejad, Delouei, & Tarokh, 2022) and the simplicity of its structure (Li, Mei, & Klausner, 2013) (Gao, Yu, Yang, Qin, & Hou, 2021), which enables the handling of complex boundary conditions effectively(Hammid et al., 2023) and practical numerical simulations (Liu, 2012). Due to its mesoscale nature, It identified a helpful method for modelling microflow(Wang, Chai, Hou, Chen, & Xu, 2018). Researchers are studying fluid flow characteristics in microchannels associated with the rapid expansion of the microelectromechanical industry. These developments include various fields of micro-heat exchanger technology, bioengineering, and microelectronic cooling solutions(Sohankar et al., 2022; Taassob, Kamali, & Bordbar, 2018).

Heat transfer improvement techniques are separated into passive and active categories; the enhancement can involve passive methods via channel configuration modification, inducing flow disruption, modifying channel shapes, and adjusting the channel's curvature. The active method can enhance the heat transfer by employing an external power source(Sadique & Murtaza, 2022). Single-phase convective heat transmission could be improved in various ways, such as augmenting the velocity gradient near a solid surface, reducing the thermal boundary layer, boosting channel thermal efficiency, and amplifying flow disruptions by placing obstructions inside the channel. Using LBM makes it easy to simulate complex fluid flow phenomena with obstructive effects in restricted flows(Islam, Ullah, & Zhou, 2021).

Various research studies have incorporated using LBM in simulating microchannel fluid dynamics and heat transmission phenomena. Zhu et al. investigate microchannel performance in the presence of ribs by analyzing the influence of rib shape. The results demonstrate that rib size, shape, and configuration significantly influenced heat transfer efficiency and performance(Zhu et al., 2021). Wang et al. examined the liquid flow under slip boundary conditions with LBM employment. The outcomes demonstrated the efficacy of slip boundary application at the microscale for simulated liquid flow(Wang et al., 2018). Kmiotek et al. examined the effect of obstacle shapes inside two-dimensional microchannels by selecting slim, triangular, and rectangular shapes(Kmiotek & Kucaba-Piętal, 2018). Bala et al. studied laminar fluid flow in a two-dimensional channel featuring three obstacles to analyze

forced convection by employing LBM(Bala, Saha, & Anwar Hossain, 2019). Hammid et al. investigated heat transmission and fluid flow of forced convection with thermal LBM in a microchannel with square baffles. The investigation incorporates temperature jump and slip velocity. The outcomes demonstrated that the rarefaction effect, barrier placement, and square obstacle selection greatly influence heat transfer and fluid flow(Hammid et al., 2023). Lori et al. investigated solid ribs with alternating vertical porous ribs. The outcomes show that porous ribs represent a higher average Nusselt number than solid ribs and significantly improved heat transfer efficiency(Lori & Vafai, 2022). Hammid et al.employed thermal LBM to examine obstacles' influence on fluid flow and numerical heat transmission. The study compared the presence and absence of obstacles. The results indicated that applying obstacle placement significantly affects the distribution of velocity and temperature, resulting in the fluid moving more slowly and accelerating cooling at the microchannel output(Hammid, Naima, Ikumapayi, et al., 2024).

Microchannels containing obstacles are frequently employed in various applications such as geothermal, heat exchangers, solar heating technics, and cooling solutions for electronic devices, and in particular, processors of computers and micromixers. When applied to microchannels, it is crucial to comprehend the impact of an obstacle's shape and geometrical features on flow (Kmiotek & Kucaba-Piętal, 2018).

The current investigation aims to investigate the influence of obstacles within microchannels and the potential for improving heat transmission and fluid flow by introducing obstacles through modifications to the microchannel configuration. The study uses the thermal lattice Boltzmann method with single relaxation time BGK and double distribution function to investigate the rarefaction effect on the overall heat transfer and fluid flow properties in the presence of three barriers imposed on lower microchannel walls. The study also aims to compare the efficiency of obstacles and analyses the effects of separated distance changes between obstacles in two-dimensional rectangular microchannel. Due to surface impact, it is essential at the micro-scale level. Temperature jump and slip velocity are considered across fluid-solid interfaces (Duan, Ma, He, Su, & Zhang, 2019) in the simulation. Also, the conditions were an extension of the microchannel walls and all obstacle boundaries of obstacles.

2. MATHEMATICAL FORMULATION

2.1 Computational Domain

Forced convection heat transfer of laminar fluid flow is analyzed in the present study. A 2D rectangular microchannel with length L and height H as depicted in the configuration shown in Fig. (1). The thermal lattice Boltzmann method (TLBM) was used with a D2Q9 lattice to model the fluid flow in a microchannel. The microchannel received a consistent input velocity (U_{in}) and temperature (T_{in}) while their walls remain fixed with the same and equal temperature T_w . Three obstacles have length and height. L_1, h respectively placed at the bottom wall, d represents separation distances between obstacles varied between $0.5.l, l, 2.l$ during the current study.

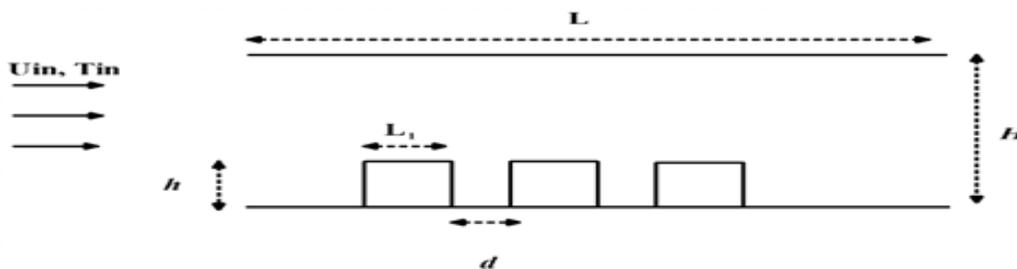


Fig 1. Microchannel configuration with obstacles

2.2 Thermal lattice Boltzmann model

The thermal lattice Boltzmann model (TLBM) with double distribution function (DDF) is one of the most current and effective LBM frameworks for managing with thermal flows (Hammid, Naima, Ikumapayi, et al., 2024). The velocity distribution function describes the macroscopic density and momentum, while the internal energy distribution function depicts the evolution of the temperature field (Sharma, Straka, & Tavares, 2020).

The BGK model with the thermal LBM was selected in the current study incorporating DDF (Hammid et al., 2023).

$$f_i(x + c_i \Delta t, t + \Delta t) - f_i(x, t) = -\frac{1}{\tau_f} [f_i(x, t) - f_i^{eq}(x, t)] \quad (1)$$

$$g_i(x + c_i \Delta t, t + \Delta t) - g_i(x, t) = -\frac{1}{\tau_g} [g_i(x, t) - g_i^{eq}(x, t)] \quad (2)$$

The distribution of density and the internal distribution function are represented by f_i, g_i , and f_i^{eq}, g_i^{eq} their equilibrium function determined in equation (3,4), τ_f, τ_g are momentum and internal energy relaxation time (Zarita & Hachemi, 2018).

$$f_i^{eq} = \omega_i \rho \left[1 + \frac{3(c_i \cdot u)}{c^2} + \frac{9(c_i \cdot u)^2}{2c^4} - \frac{3(u \cdot u)}{2c^2} \right]; \quad (3)$$

$$g_0^{eq} = -\omega_0 \rho \varepsilon \frac{3(u \cdot u)}{2c^2}; \quad (4)$$

$$g_{1,2,3,4}^{eq} = \omega_{1,2,3,4} \rho \varepsilon \left[1.5 + \frac{3(c_i \cdot u)}{2c^2} + \frac{9(c_i \cdot u)^2}{2c^4} - \frac{3(u \cdot u)}{2c^2} \right]; \quad (5)$$

$$g_{5,6,7,8}^{eq} = \omega_{5,6,7,8} \rho \varepsilon \left[3 + \frac{6(c_i \cdot u)}{c^2} + \frac{9(c_i \cdot u)^2}{2c^4} - \frac{3(u \cdot u)}{2c^2} \right] \quad (6)$$

A two-dimensional square lattice with nine velocities, with the central node related to eight surrounding nodes, is displayed in Fig. (2). The center and adjacent nodes transmit momentum and energy through distribution functions throughout the streaming-collision procedure. (Samanta, Chattopadhyay, & Guha, 2022).

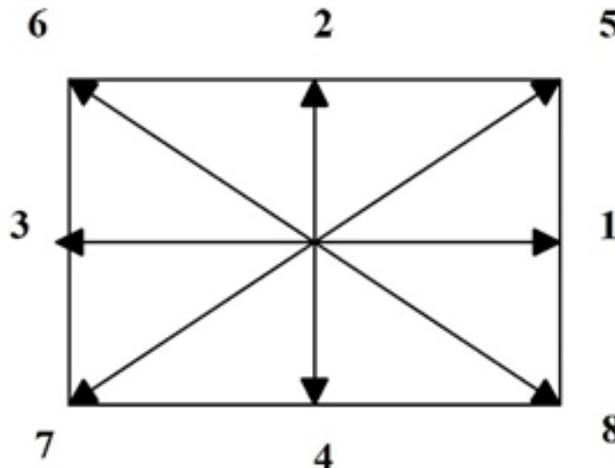


Fig 2. D_2Q_9 model

For D_2Q_9 , the c_i indicates discrete velocity vector for all lattice nodes in space and is expressed as (Li, Mei, & Klausner, 2017)

$$c_i = (0,0) \quad (i = 0) \tag{7}$$

$$c_i = \left[\cos \frac{(i-1)\pi}{2}, \sin \frac{(i-1)\pi}{2} \right] c \quad i = 1,2,3,4 \tag{8}$$

$$c_i = \left[\cos \frac{(i-1)\pi}{4}, \sin \frac{(i-1)\pi}{4} \right] c\sqrt{2} \quad i = 5,6,7,8 \tag{9}$$

The weighting coefficient for the D2Q9 discrete velocity model is determined by:

$$\omega_i = \frac{4}{9} \text{ for } i = 0; \omega_i = \frac{1}{9} \text{ for } i = 1,2,3,4 \quad \omega_i = \frac{1}{36} \text{ for } i = 5,6,7,8 \tag{10}$$

From the distribution function, can be derived the macroscopic quantity for the determination of density, velocity, and temperature (Hammid, Naima, Ikumapayi, et al., 2024):

$$\rho = \sum_i f_i; \rho \vec{u} = \sum_i f_i \vec{c}_i; \rho \varepsilon = \rho RT = \sum_i g_i \tag{11}$$

2.3 Numerical methodology and validation procedures

The current study utilized a Python coding environment to simulate heat transport and fluid dynamics in a microchannel. To guarantee the accuracy of the simulation of outcomes, the velocity profiles produced by LBM were compared to those from Direct Simulation Monte Carlo (DSMC) by (Roohi & Darbandi, 2009). This comparison illustrates the validity and significance of the existing numerical method for modelling forced convection heat transfer within a microchannel (Fig. 3).

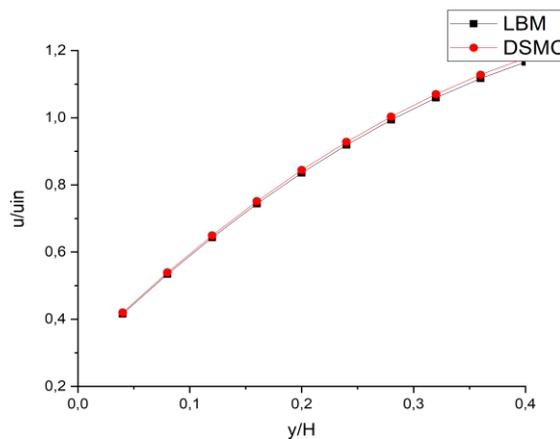


Fig 3. LBM and DSMC comparison with velocity profile for $Kn = 0.113$ with (Roohi & Darbandi, 2009)

3. RESULT AND DISCUSSION

3.1 Investigating the effect of separation distance change

Figure (4) shows the velocity variation for fixed Knudsen number $Kn = 0.03$ in the function of the separation distance between obstacles d . The results show velocity augmentation along the microchannel until the microchannel center reaches a peak value, then decreases toward the end of the microchannel. According to the results, the middle of the microchannel gives maximum values while the low values are close to the walls, similar to the previous study (Hammid, Naima, Ikumapayi, et al., 2024; Hammid et al., 2023).

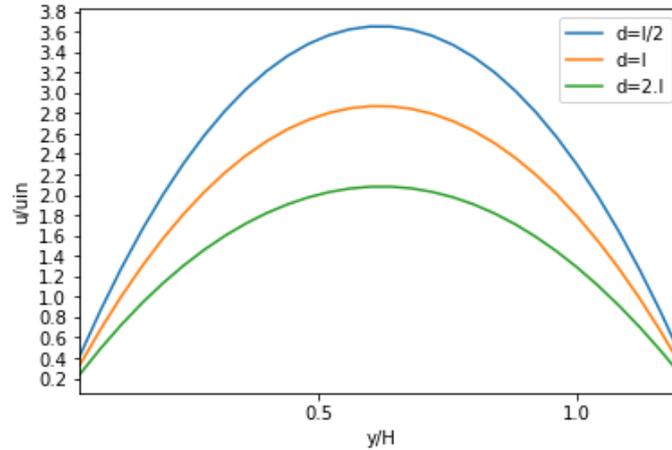


Fig 4. Velocity distribution in function of distance separation $Kn=0.03$

Furthermore, raising separation distances aids significantly in velocity dropping ($d = 2.1$) and reaches a maximum value at microchannel center. Results show that increased distance between obstacles decreases velocity at microchannel exit. It is worth considering this observation in microfluidic device design.

Cooling is essential in several technologies, such as microelectronics and nanoelectromechanical components. Due to the rising need for microscale manufacturing, an increasing focus on improving heat transfer mechanisms (Behnampour et al., 2017). The temperature distribution along the microchannel, characterized by a Knudsen number of 0.03, varies as a function of the distance between obstacles depicted in Fig. (5). The results show a notable temperature drop observed throughout the microchannel. At the centre of the microchannel, temperatures with low values are achieved. As the distance between barriers increases, there is a temperature rise. However, the rate of reduction slows down. The scenario with the smallest distance between obstacles results in lower temperature values compared to those observed in the case with a greater separation distance. According to outcomes, it can be concluded that decreasing the separation distance between obstacles correlates with a reduction in temperature, highlighting the significant impact of barriers on temperature distribution. Moreover, temperatures adjacent to the wall of the microchannel exhibit an increase along with a rise in distance separation.

3.2 Slip velocity

Rarefaction effects are considered an essential factor in microscale systems represented by Knudsen number determined by mean free path to characteristic length ratio (Hammid, Naima, Alqahtani, et al., 2024). Additionally, slip velocity has a significant impact on the microchannel flows. Fig. (6), represents the axial change of slip velocity across the microchannel's bottom wall. A rapid reduction in slip velocity

is displayed at microchannel entrance, but it shows increases at obstacle level. This augmentation also drops with Knudsen numbers rising and slip velocity value decreasing respectively to increasing separation distances between obstacles, then slip velocity maintains constant until the end of microchannel. The findings reveal a noticeable dropping in slip velocity along microchannel length as Knudsen numbers rise.

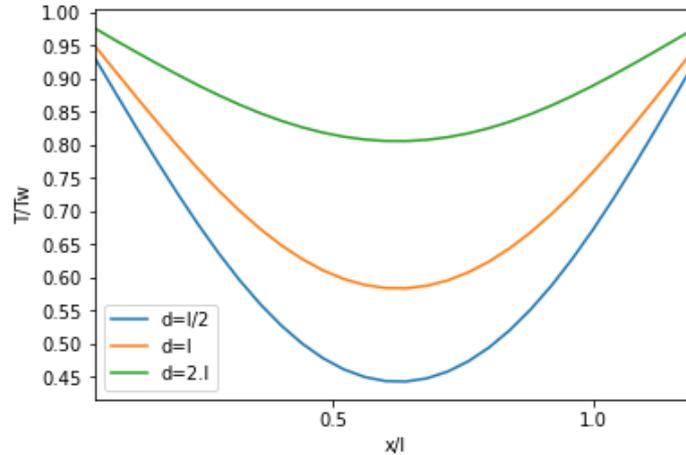


Fig 5. Temperature distribution in function of distance separation $Kn = 0.03$

Also, it is noteworthy that the maximum slip velocity is attained at the obstacle level. Moreover, the rarefaction effect significantly impacts microchannel output and obstacle level compared to microchannel entrance.

3.3 Temperature jump

Temperature jump variation in function of Knudsen numbers $Kn = 0.02, 0.04, \text{ and } 0.06$ is depicted in Fig. (7). At the entrance region and across the developing area, the temperature increases rapidly and remains constant until the microchannel ends. In addition, the jumping of temperature is impacted by the augmentation of Knudsen numbers, which shows a decrease. From the result, the rarefaction effect along the microchannel wall was little on temperature jump compared to the obstacles zone, which was insignificant.

4. CONCLUSION

A rectangular microchannel with multiple obstacles was studied using the thermal lattice Boltzmann method with a double distribution function and BGK approximation. The study incorporates conditions of temperature jump and slip velocity along microchannel walls. The separation distance between obstacles was taken into consideration during the study. The study delves deeper into the varying separation distance between obstacles along the microchannel wall to investigate their impact on fluid flow dynamics and heat transmission characteristics. Moreover, the analysis explores the impact of separation distance by providing insights into the complex interactions between microscale geometry and heat transmission in rarefied flows. The key findings of the study can be presented as follows:

- Raising separation distances between barriers significantly aids in velocity dropping, which can benefit applications requiring quick cooling processing.
- The dropping of separation distance between obstacles reduces temperature, highlighting the significant impact of obstacles on temperature distribution, leading to transfer enhancement efficiency.

- Across the microchannel length, the findings indicate a substantial reduction in slip velocity with Knudsen number augmentation.
- The rarefaction effect shows a little to insignificant impact on temperature jump.

The study outcomes could assist in developing cooling solutions for various technologies, including nanoelectromechanical and microelectronics systems. The proposed configuration may also improve the performance and design of microfluidic devices. By optimizing fluid flow and heat transmission, these outcomes could contribute significantly to boosting the thermal management of advanced technological systems.

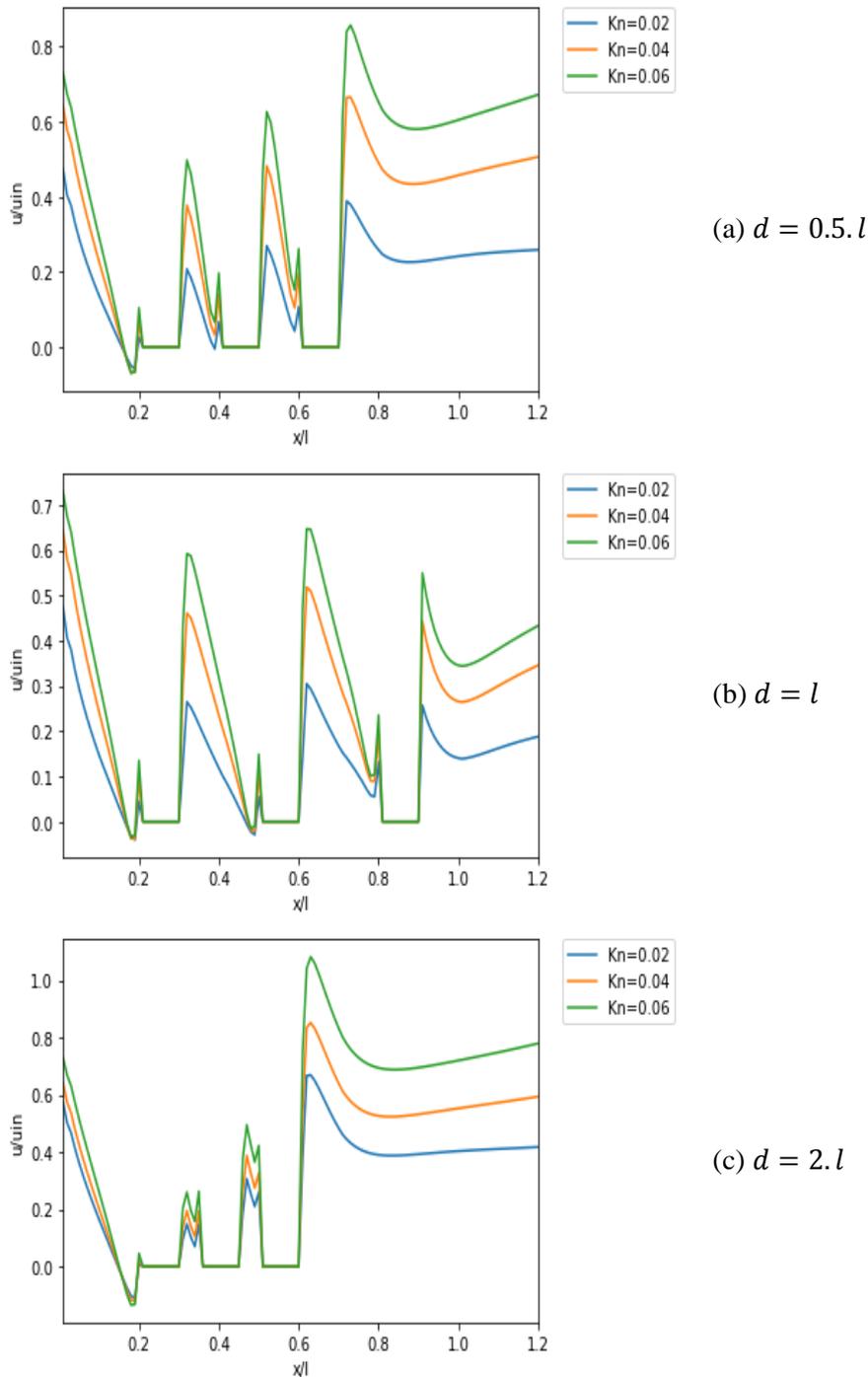


Fig 6. Slip velocity variation with Knudsen numbers

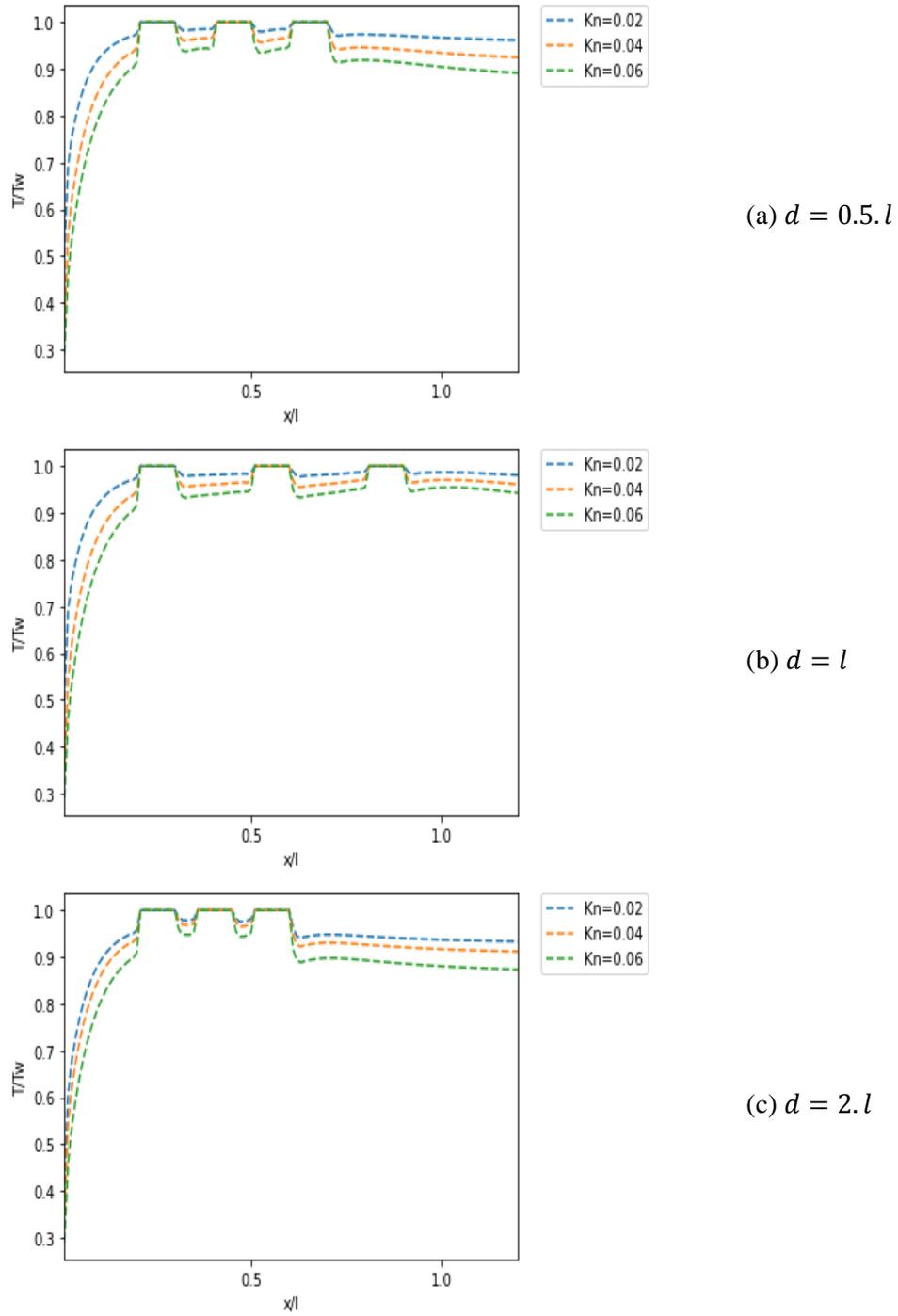


Fig 7. Temperature jump variation with Knudsen numbers

NOMENCLATURE

f	Density momentum distribution function	H	Non-dimensional height of microchannel
g	Internal energy distribution function	L	Non-dimensional length of microchannel
T_{in}	Inlet temperature (K)	d	Separation distance
T_w	Wall temperature (K)	h	Obstacle height
u_{in}	Inlet velocity (ms^{-1})	L_1	Obstacle length

REFERENCES

- Abaszadeh, M., Safavinejad, A., Amiri, H., & Amiri Delouei, A. (2022). A direct-forcing IB-LBM implementation for thermal radiation in irregular geometries. *Journal of Thermal Analysis and Calorimetry*, 147(20), 11169-11181. Doi:10.1007/s10973-022-11328-1
- Afra, B., Delouei, A. A., & Tarokh, A. (2022). Flow-Induced Locomotion of a Flexible Filament in the Wake of a Cylinder in Non-Newtonian Flows. *International Journal of Mechanical Sciences*, 234, 107693. Doi: <https://doi.org/10.1016/j.ijmecsci.2022.107693>
- Afra, B., Karimnejad, S., Delouei, A. A., & Tarokh, A. (2022). Flow control of two tandem cylinders by a highly flexible filament: Lattice spring IB-LBM. *Ocean Engineering*, 250, 111025. Doi: <https://doi.org/10.1016/j.oceaneng.2022.111025>
- Bala, S. K., Saha, L. K., & Anwar Hossain, M. (2019). Simulation of forced convection in a channel containing three obstacles over backward and forward facing steps by LBM. *International Journal of Applied and Computational Mathematics*, 5(2), 1-19. Doi: <https://doi.org/10.1007/s40819-019-0622-2>
- Behnampour, A., Akbari, O. A., Safaei, M. R., Ghavami, M., Marzban, A., Shabani, G. A. S., & Mashayekhi, R. (2017). Analysis of heat transfer and nanofluid fluid flow in microchannels with trapezoidal, rectangular and triangular shaped ribs. *Physica E: Low-Dimensional Systems and Nanostructures*, 91, 15-31. Doi: <https://doi.org/10.1016/j.physe.2017.04.006>
- Duan, Z., Ma, H., He, B., Su, L., & Zhang, X. (2019). Pressure drop of microchannel plate fin heat sinks. *Micromachines*, 10(2), 80. Doi: <https://doi.org/10.3390/mi10020080>
- Gao, Y., Yu, Y., Yang, L., Qin, S., & Hou, G. (2021). Development of a coupled simplified lattice Boltzmann method for thermal flows. *Computers & Fluids*, 229, 105042. Doi: <https://doi.org/10.1016/j.compfluid.2021.105042>
- Ghadirzadeh, S., & Kalteh, M. (2017). Lattice Boltzmann simulation of temperature jump effect on the nanofluid heat transfer in an annulus microchannel. *International Journal of Mechanical Sciences*, 133, 524-534. Doi: <https://doi.org/10.1016/j.ijmecsci.2017.09.013>
- Hammid, S., Naima, K., Alqahtani, S., Alshehery, S., Oudah, K. H., Ikumapayi, O. M., & Menni, Y. (2024). Laminar rarefied flow analysis in a microchannel with H₂O-Cu nanofluid: A thermal lattice Boltzmann study. *Modern Physics Letters B*, 38(03), 2450006. Doi: <https://doi.org/10.1142/s0217984924500064>
- Hammid, S., Naima, K., Ikumapayi, O. M., Kezrane, C., Liazid, A., Asad, J., . . . Menni, Y. (2024). Overall Assessment of Heat Transfer for a Rarefied Flow in a Microchannel with Obstacles Using Lattice Boltzmann Method. *Comput. Model. Eng. Sci*, 138, 273-299. Doi: <https://doi.org/10.32604/cmesci.2023.028951>
- Hammid, S., Naima, K., Menni, Y., Kezrane, C., Liazid, A., Al-Dujaili, A. Q., . . . Shoja, S. J. (2023). Advanced numerical analysis of convective rarefied flows in microchannels: Studying the impact of multiple obstacle arrangements through LBM modeling. *Modern Physics Letters B*, 37(27), 2350099. Doi: <https://doi.org/10.1142/s0217984923500999>
- Islam, S.-U., Ullah, N., & Zhou, C. Y. (2021). Numerical analysis of blockage effects on the flow between parallel plates by using lattice Boltzmann method. *Canadian Journal of Physics*, 99(6), 399-411. Doi: <https://doi.org/10.1139/cjp-2020-0240>

- Kmiotek, M., & Kucaba-Piętal, A. (2018). Influence of slim obstacle geometry on the flow and heat transfer in microchannels. *Bulletin of the Polish Academy of Sciences: Technical Sciences*(2) Doi: 10.24425/119064
- Li, L., Mei, R., & Klausner, J. F. (2013). Boundary conditions for thermal lattice Boltzmann equation method. *Journal of Computational Physics*, 237, 366-395. Doi: <https://doi.org/10.1016/j.jcp.2012.11.027>
- Li, L., Mei, R., & Klausner, J. F. (2017). Lattice Boltzmann models for the convection-diffusion equation: D2Q5 vs D2Q9. *International Journal of Heat and Mass Transfer*, 108, 41-62. Doi: <https://doi.org/10.1016/j.ijheatmasstransfer.2016.11.092>
- Liu, Y. (2012). A lattice Boltzmann model for blood flows. *Applied Mathematical Modelling*, 36(7), 2890-2899. Doi: <https://doi.org/10.1016/j.apm.2011.09.076>
- Lori, M. S., & Vafai, K. (2022). Heat Transfer and Fluid Flow Analysis of Microchannel Heat Sinks with Periodic Vertical Porous Ribs. *Applied Thermal Engineering*, 118059. Doi: <https://doi.org/10.1016/j.applthermaleng.2022.118059>
- Roohi, E., & Darbandi, M. (2009). Extending the Navier–Stokes solutions to transition regime in two-dimensional micro-and nanochannel flows using information preservation scheme. *Physics of fluids*, 21(8), 082001. Doi: <https://doi.org/10.1063/1.3177351>
- Sadique, H., & Murtaza, Q. (2022). Heat transfer augmentation in microchannel heat sink using secondary flows: A review. *International Journal of Heat and Mass Transfer*, 194, 123063. Doi: <https://doi.org/10.1016/j.ijheatmasstransfer.2022.123063>
- Samanta, R., Chattopadhyay, H., & Guha, C. (2022). A review on the application of lattice Boltzmann method for melting and solidification problems. *Computational Materials Science*, 206, 111288. Doi: <https://doi.org/10.1016/j.commatsci.2022.111288>
- Sharma, K. V., Straka, R., & Tavares, F. W. (2020). Current status of Lattice Boltzmann Methods applied to aerodynamic, aeroacoustic, and thermal flows. *Progress in Aerospace Sciences*, 115, 100616. Doi: <https://doi.org/10.1016/j.paerosci.2020.100616>
- Sohankar, A., Joulaei, A., & Mahmoodi, M. (2022). Fluid flow and convective heat transfer in a rotating rectangular microchannel with various aspect ratios. *International Journal of Thermal Sciences*, 172, 107259. Doi: <https://doi.org/10.1016/j.ijthermalsci.2021.107259>
- Taassob, A., Kamali, R., & Bordbar, A. (2018). Investigation of rarefied gas flow through bended microchannels. *Vacuum*, 151, 197-204. Doi: <https://doi.org/10.1016/j.vacuum.2018.02.021>
- Wang, K., Chai, Z., Hou, G., Chen, W., & Xu, S. (2018). Slip boundary condition for lattice Boltzmann modeling of liquid flows. *Computers & Fluids*, 161, 60-73. Doi: <https://doi.org/10.1016/j.compfluid.2017.11.009>
- Zarita, R., & Hachemi, M. (2018). Numerical investigation and analysis of heat transfer enhancement in a microchannel using nanofluids by the lattice Boltzmann method. *Frontiers in Heat and Mass Transfer (FHMT)*, 12. Doi: <https://doi.org/10.5098/hmt.12.5>
- Zhu, Q., Jin, Y., Chen, J., Su, R., Zhu, F., Li, H., . . . Cui, Y. (2021). Computational study of rib shape and configuration for heat transfer and fluid flow characteristics of microchannel heat sinks with fan-shaped cavities. *Applied Thermal Engineering*, 195, 117171. Doi: <https://doi.org/10.1016/j.applthermaleng.2021.117171>