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Analysis of PV/Wind systems by integer linear programming

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Abstract - *This paper presents a mathematical formulation for integrating renewable energy sources in order to build up economical hybrid energetic systems in the case where each type of energy are only available as specific units. For instance, we may need to combine photovoltaic panels and wind turbines with specific capacities to meet an energetic demand in a specific site with a lowest cost. Therefore, determining the optimal energy to be installed leads of determining the number of units from each source. This problem is formulated as an integer linear program where the objective function to be minimized is the initial capital investment and where the decision variables are the numbers of units which should be pure integer numbers. The proposed method is generic since it is directly adaptive to various sites, demands, and different type of renewable energy sources. To illustrate our approach, we provide some examples and scenarios combining PV and wind turbines units.*

Résumé - *Cet article présente une formulation mathématique pour l'intégration de sources d'énergie renouvelables afin de mettre en place des systèmes hybrides énergétiques économiques dans le cas où chaque type d'énergie ne sont disponibles que sous forme d'unités spécifiques. Par exemple, nous pourrions avoir besoin de combiner des panneaux photovoltaïques et des éoliennes avec des capacités spécifiques pour répondre à une demande énergétique dans un site spécifique avec un moindre coût. Par conséquent, la détermination de l'énergie optimale pour être installée conduit à connaître le nombre d'unités provenant de chaque source. Ce problème est formulé comme un programme linéaire en nombres entiers, où la fonction objective à minimiser est l'investissement initial et où les variables de décision sont les nombres d'unités qui devraient être des nombres entiers purs. La méthode proposée est générique, car elle est directement adaptée à des divers sites, de demandes, et de différents types de sources d'énergie renouvelables. Pour illustrer notre approche, nous donnons quelques exemples et des scénarios combinant les unités photovoltaïques et les éoliennes.*

Keywords: Hybrid renewable energetic systems - Integration of renewable energy - Optimal energy distribution - Photovoltaic - Wind turbine Integer programming.

1. INTRODUCTION

Hybrid renewable energetic systems are systems that integrate more than one renewable energy sources. As they are time, environment and site dependant, one expects that their judicious and complementary combination may overcome some limitations which are inherent to every individual system used alone. Hybrid systems may also reduce the need for energy storage which is very costly and space consuming.

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In this context, the topic of optimizing the integration of renewable energy sources in a complementary way is a very interesting but a challenging one both scientifically and technologically.

The general problem of combining renewable energy sources we are considering can be stated as follows: in a specific site, given the capacities of some renewable energy sources and an energetic demand, we need to determine the optimal repartition of these energies that meets the demand. There have been few proposed approaches to solving this problem with and without taking into account energy storage systems such as batteries, diesel engines, hydrogen, etc.

Among these approaches, we can notice linear programming, dynamic programming, genetic algorithms techniques, etc [1-4, 8]. Some software for analysis and optimization of hybrid energetic systems has been developed and are actually largely used such as Homer, Somes, Rapsim, Sosim, etc. [5-7].

In [9], the authors revises some relevant papers concerning the simulation and optimization techniques, as well as the tools existing that are needed to simulate and design stand-alone hybrid systems for the generation of electricity.

However, there is situations where a renewable energy source is available only as specific units. For instance, manufacturers may provide photovoltaic panels as units with a specific capacity characterized mainly by its generated pick power.

Manufacturer also may provide wind turbines as units with specific size and then with a specific capacity according to the wind characteristics in the considered site. As a consequence, the hybrid system may be composed of a certain number of photovoltaic panels and of a certain number of wind turbines.

Therefore, determining the optimal energy to be installed leads to determining the number of units from each source which are required to meet the specified energetic demand.

This type of problems that require integer numbers of renewable energetic units has been formulated in many ways but the most used approach seeks to minimize the Loss of Power Supply Probability (LPSP).

In [10], the authors consider the case where they have photovoltaic panels and wind turbines as specific units. Thus, the problem consists of determining the numbers of PV panels, wind turbines, and batteries. Their formulation leads to a non linear integer programming problem. They solve the problem by using the LPSP in the framework of ant systems by minimizing the initial capital investment.

In [11], an optimal sizing method for stand-alone hybrid solar–wind system with LPSP technology is adopted by using genetic algorithm. It consists of determining, among other elements, the number of PV and the number of wind turbines.

In [12], a hybrid system is used and the formulation determines the number of micro-hydro, PV, wind turbines, and batteries by minimizing the life cycle cost.

In [13], an economical approach minimizing the cost is used to determine the optimum battery capacity, together with optimum number of PV modules and wind turbines.

In [14], the LPSP model is used and the lowest Levelised Cost of Energy is considered as the economical optimal configuration. The optimal configuration determine the number of PV modules, the capacity of wind turbine, and the capacity of battery bank.

Following the simple but attractive formulation using linear programming to model and solve the problem of sources repartition without taking into account the storage issue [1, 8], we address the problem of optimal distribution of renewable energetic hybrid systems where the energies provided by each source are constituted of specific units.

We formulate the problem as an integer linear programming one by minimizing the initial capital investment under constraints such as ensuring the annual required energetic demand. The formulation requires the estimation of the annual energy that should be provided by each renewable energy unit as well as its cost. To illustrate our analysis, we provide examples combining PV and wind turbines units.

2. EXPERIMENTAL RENEWABLE ENERGY SYSTEMS

The aim of our study consists of satisfying energetic demands in specific sites by integrating in an economical way the available renewable energetic sources. We assume that the energy provided by each source are constituted of units.

Fig.1 shows some experimental prototypes of various renewable energy sources (units) which are available or have been built up in the LATA at University of Constantine.

From left to right, picture (a) shows a photovoltaic panel, picture (b) shows a small wind turbine, picture (c) shows a dish for thermal solar energy, and figure (d) shows a bio-digester for methane production from waste-water and animal manure.

a- b- c- d-

Fig. 1 (a- b- c- d-): Units of some renewable energy systems

While the approach is generic and can be extended to many renewable energy sources, we provide the analysis of a small hybrid system composed of photovoltaic panels and wind turbines. An example is reported to a site which is situated at Constantine city in Algeria with respectively a latitude and a longitude of: 36° 21' 54" N / 6° 36' 53" E.

2.1 Estimation of annual photovoltaic energy

The energy produced by a photovoltaic panel over a period of time depends on many factors but mainly on the surface of the panel, its pick power, the incident irradiance which depends on its location and on season and hour of the day, weather conditions, shadowing, etc. The energy produced by a photovoltaic panel over a period of time can be estimated by the following expression [15-17]:

$$E_{ph} = P_r \times F_0 \times (G_{eff} / G_0) \times P_c \qquad (1)$$

where:

Pr, Performance ratio (average value 0.72or 0.75) with 0.75 for an optimal orientation of the panel

F_0, Factor that accounts for losses

G_{eff}, Effective annual incident Irradiance

G_0, Irradiance under standard conditions (1 000 W/m^2)

P_c, Nominal power under standard conditions as provided by the manufacturer.

We can estimate the annual power produced by one photovoltaic panel according to (1) but we can also use an empirical rule, in a sunny zone like our site, by considering that one Watt of peak power produces an annual energy of about 2 kWh [17].

Since PV modules which are available at our laboratory are made of Silicon crystalline cells and have a peak power P_c = 33 W. So, a single photovoltaic panel (one unit) produces about: 66 kWh/year. The surface of a panel is: 0.42 m^2.

2.2 Estimation of annual wind turbine energy

To estimate the power generated by a wind turbine, we use the following expression [18]:

$$P_{Turbine} = \frac{1}{2} C_P \times \rho \times \pi \times R^2 \times V^3 \quad (2)$$

where V, is the wind speed, R, the blade radius, C_p, the operating efficiency factor, ρ, the density of air at sea level, which is about 1.2 kg/m^3. The height of the turbine may improve the performances of the wind turbine by increasing the wind speed according to the expression (4):

$$V = V_0 \times (h / h_0)^{\alpha} \quad (3)$$

h, Hub height and h_0, Reference height which is usually 10 m

V, Wind speed at hub height h, and V_0, Wind speed at the reference height (10m), H_0 and α, Power law exponent, which is usually taken as 1/7.

In the location where the measures have been obtained, the average value of the wind speed over 27 years is about 2.5 m/s which means that the site is not very interesting for wind turbine exploitation. Nevertheless, there are some elevated site which are more windy.

Thus, if we consider our experimental small wind turbine built up in our laboratory with a blade diameter of 2 m and an operating efficiency factor C_p = 20% at a reference average wind speed of 4 m/sec, then elevated only at 30 m height, we obtain a wind speed of about 4.67 m/s.

This gives an estimated power of about: $P_{Turbine} \cong 38$ Watts. If we assume that the wind turbine produces the expected power of 2190 hours per year, then it would produce about: $E_w \cong 84$ kWh / year. The swept area of the wind turbine is 3.14 m^2.

However, since the power is varying with respect to the environment conditions (solar radiation, wind speed, temperature,.), there is no way to avoid energy storage or energy generation to compensate the lack of energy for certain moments.

3. MODELING HYBRID SYSTEMS IN TERMS OF INTEGER LINEAR PROGRAMMING

3.1 General formulation

The problem of optimal reparation of renewable energies can be stated as follows. Consider some renewable energy sources such as: Photovoltaic panel X_{ph} with limit L_{ph} and unit cost C_{ph}; Thermal solar panel X_{ph} with limit S_{ph} and unit cost C_{ph}; Wind turbine $X_{éo}$ with limit $S_{éo}$ and unit cost $C_{éo}$; Biomass X_{bi} with limit S_{bi} and unit cost C_{bi}; and the demand by D.

If there were no more constraints, this problem can be formulated as a linear programming one [1, 8]. But, since the energy of each source is constituted from specified units, this leads to add additional constraints on the decision variables that should be pure integer numbers.

If we consider the case of two types of energy, say photovoltaic and wind turbine. We have elementary photovoltaic units and elementary wind turbines units. The problem consists of determining the number of photovoltaic panel as well as the number of wind turbines.

Therefore, the problem can be converted into an optimization program known as integer linear programming problem that requires pure integer solutions for decision variables N_1 and N_2.

An initial general formulation consists of minimizing a cost function Z_T while satisfying the demand D. The unit costs of each renewable unit is C_i and its annual energy production is E_i. The problem can be expressed as follows:

$$\begin{cases} \min(Z_T) = \sum_i C_i \times N_i \\ \sum_i E_i \times N_i = D \qquad\qquad i = 1, 2, 3, \ldots \\ N_i \geq 0 \\ N_i \quad \text{Integers} \end{cases} \tag{4}$$

Compared to linear programming problems, integer linear programming problems are more difficult to solve and have specific techniques for their resolution.

In the last twenty years or so, the most effective technique has been based on dividing the problem into a number of smaller problems in a method called branch and bound [19].

For simpler case with two decision variables, graphical representation can help to analyze and solve the problem if a solution exists.

There exist integer linear programming solvers such as Lindo [20], lp_solve [21], etc.

3.2 An example of formulation

Let's consider the following example of a small PV/Wind hybrid system. The estimated energy produced by a photovoltaic panel (one unit) is $E_{ph} = 66 \text{ kWh / year}$ and the estimated energy produced by a wind turbine (one unit) is $E_w = 83 \text{ kWh / year}$.

If we assume a cost of 4 \$ per Watt of peak power, then the unit cost of a photovoltaic panel is around $C_1 = 130\ \$$.

On the other hand, the reduced cost of the type of wind turbine which has been built up (except the electrical part) at our laboratory can be estimated at about $C_2 = 100\ \$$. This leads to 2 \$/Watt.

For simplicity, we may only consider in this analysis the investment for capital cost of the hybrid system which may involve the number of PV panels (N_1) and the number of wind turbines (N_2). This capital cost which is the objective function has to be minimized, therefore:

$$\min\ Z_T = C_1 \times N_1 + C_2 \times N_2 \quad (\$) \tag{5}$$

If we assume an electrical demand D (kWh/year). Thus, to meet this demand, we need to use a certain number of PV panels and a certain number of wind turbines.

This leads to satisfy the following constraint:

$$E_{ph} \times N_1 + E_w \times N_2 = D \qquad \text{(kWh/year)} \tag{6}$$

Note that, in integer linear programming problems, it may be difficult to find out a feasible solution that meets this equality constraint. So, one needs to extend the field of feasible solutions by replacing the equality (6) by the inequality constraint such as:

$$E_{ph} \times N_1 + E_w \times N_2 \geq D \qquad \text{(kWh/year)} \tag{7}$$

Moreover, since the number of PV panels (N_1) and the number of wind turbines (N_2) should be integers, therefore, the problem can be basically expressed as a pure integer linear program as follows:

$$\begin{cases} \min(Z_T) = C_1 \times N_1 + C_2 \times N_2 \\ \text{Subject to} \\ \sum_i E_i \times N_i = D \qquad\qquad i = 1, 2, 3, \ldots \\ N_1, N_2 >= 0 \\ N_1, N_2 \quad \text{Integers} \end{cases} \tag{8}$$

4. CASES ANALYSIS

4.1 Analysis of an example

Let's assume $C_1 = 130\ \$$ and $C_2 = 100\ \$$, then the objective function Z_T can be expressed as follows:

$$\min\ Z_T = 130 \times N_1 + 100 \times N_2 \tag{9}$$

Let's assume (9) and $E_W = 83\ \text{kWh/year}$ and an electrical demand D of about 3000 kWh/year. Thus, the problem can be expressed as a pure integer linear program as follows:

$$\begin{cases} \min Z_T = 130 \times N_1 + 100 \times N_2 \\ \quad \text{Subject to} \\ 66 \times N_1 + 84 \times N_2 = 3000 \\ \quad N_1, N_2 >= 0 \\ \quad N_1, N_2 \quad \text{Integers} \end{cases} \tag{10}$$

The solution gives an objective function value = 3880 \$ with $N_1 = 6$ photovoltaic panel and $N_2 = 31$ wind turbines.

To illustrate the search of solutions in integer linear programming, we present and discuss the graphical solution of the previous program. The graphical analysis is presented in Fig. 2 where the x-axis represents N_1 and the y-axis represents N_2. The red line represents the equality constraint ($66 \times N_1 + 84 \times N_2 = 3000$).

The blue lines represents the objective function ($Z = 130 \times N_1 + 100 \times N_2$) which is parameterized by the value Z. So, by increasing the value of Z while moving the blue line in the direction of the arrows, we seek to interest the red line in a point where both N_1 and N_2 should be integers. If there are many points that meet this condition of integrality, we should select the point which corresponds to the minimal value of Z.

In the example under consideration, the optimal value corresponds to the point M where $Z = 3880$ \$ with $N_1 = 6$ and $N_2 = 31$ which have been already given by the numerical method.

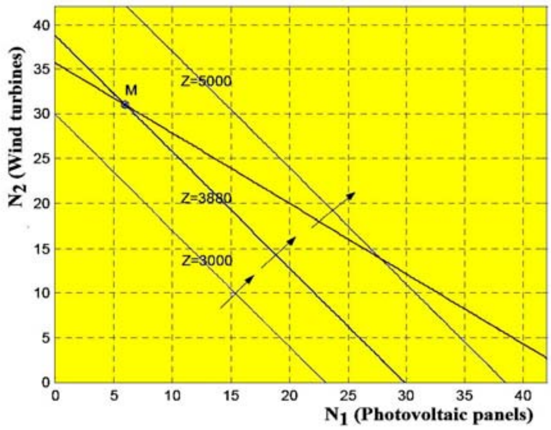

Fig. 2: Graphical analysis of a solution in integer linear programming

4.2 Sensitivity of the solution with respect to an annual energy unit

Let's analyze the sensitivity of the solution (if it exists) of the integer program with the equality constraint (10) with respect to a small change in the coefficient characterizing the annual energy production of a wind turbine unit E_W. **Table 1** shows the sensitivity of the solution with respect to E_W.

Table 1: Sensitivity of the solution w.r.t. E_W

E_W (Cost - wind turbine)	Z ($) (Objective function)	N_1 (PV panels)	N_2 (Wind turbine)
80	4700	20	21
81	4480	16	24
82	5540	38	6
83	No solution	//	//
84	3880	6	31
85	5100	30	12

From **Table 1**, we may notice how much, under some circumstances, the integer program can be very sensitive to a small change with respect to an energy unit such as E_{ph} or E_W. As a result, a supervision of the solution as well as an analysis of its sensitivity have to be performed by the designer before deciding which solution to choose.

Within the form (10), if $E_W = 83\,\text{kWh/year}$, the problem has no integer solution. This means that, graphically, there is no intersection between the equality constraint and the objective function that corresponds to a point with integer coordinates. However, we may find a solution to this problem if we replace the equality constraint by the corresponding inequality as follows:

$$\begin{cases} \min Z_T = 130 \times N_1 + 100 \times N_2 \\ \text{Subject to} \\ 66 \times N_1 + 84 \times N_2 \geq 3000 \\ N_1, N_2 >= 0 \\ N_1, N_2 \quad \text{Integers} \end{cases} \tag{11}$$

The solution gives an objective function value $Z = 3700$ \$ with $N_1 = 0$ photovoltaic panel and $N_2 = 37$ wind turbines. According to this result, the system will be only composed of wind turbines because a wind turbine is cheaper and produces more energy than a photovoltaic panel.

4.3 More renewable energy sources

This example generalizes the proposed model to problems with three renewable energy sources such as photovoltaic panel, wind turbine and photo-thermal systems.

N_1, N_2, N_3 are respectively the numbers of photovoltaic panel, wind turbines and photo-thermal panels.

$$\begin{cases} \min Z_T = 130 \times N_1 + 100 \times N_2 + 70 \times N_3 \\ \text{Subject to} \\ 66 \times N_1 + 83 \times N_2 + 25 \times N_3 \geq 3000 \\ N_1, N_2, N_3 \geq 0 \\ N_1, N_2, N_3 \quad \text{Integers} \end{cases} \tag{12}$$

The solution of this program outputs the value of the objective function $Z = 4300$ \$, and $N_1 = 12$ photovoltaic panel; $N_2 = 26$ wind turbines; $N_3 = 2$ photo-thermal panels.

4.4 Adding more constraints

The discussed examples above are basic ones but more constraints can be included. These constraints may stem from energetic, space and number of units limitations.

In the following example, we want to limit the space occupied by wind turbines by reducing their numbers below six ($N_2 < 6$). The problem can be formulated as follows:

$$\begin{cases} \min Z = 130 \times N_1 + 100 \times N_2 + 70 \times N_3 \\ \quad \text{Subject to} \\ 66 \times N_1 + 83 \times N_2 + 25 \times N_3 = 3000 \\ \quad N_2 < 6 \\ \quad N_1, N_2, N_3 \geq 0 \\ \quad N_1, N_2, N_3 \quad \text{Integers} \end{cases} \tag{13}$$

The solution of this program outputs the value of the objective function $Z_T = 5820$ and $N_1 = 35$ photovoltaic panels; $N_2 = 5$ wind turbines; $N_3 = 11$ photo-thermal panels.

5. CONCLUSION

We have presented a formulation for optimally combining renewable energy sources in order to build up economical hybrid energetic systems in case where these energies are only available as specific units.

As a consequence, determining the optimal energy to be installed leads of determining the number of units from each source. The problem has been modelled as an integer linear programming problem where decision variables should be pure integer numbers.

The developed model directly relates the group of input parameters which are the estimated power and annual energy production of each renewable energy source. These parameters depend on environmental conditions (wind speed, solar irradiation, temperature), on geographical characteristics of the site (latitude, longitude, altitude), and on the design characteristics of each energetic system (rotor diameter and hub height of the wind turbine, peak power and panel orientation, etc.).

So, given an estimation of the annual energy production and the cost of each unit of the renewable energy source as well as an annual demand, the program outputs (if possible) the number of units from each source to meet the demand while satisfying all the constraints.

The method is thus generic since it is directly adaptive to various sites, demands, and different energy sources.

To illustrate our approach, we have provided some examples of simulation and scenarios combining PV and wind turbines sources. However, solving integer programs is much difficult than solving linear programs. So, the designer may be involved for proposing and supervising various scenarios in order to find out and choose realistic solutions.

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