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# Modeling a fertilisant dynamic transformation in 'soil-plant' system

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**Abstract** - *The application of fertilizers is one of the conditions for obtaining the resistant cultures. Effectiveness of contribution fertilizers such as phosphorus, the nitrogen, etc…depends on many factors to knowing: properties of the soil, the request of the plants out of fertilizers (phosphorescence for example) and also on the form and the application method of fertilizers. Several results show that rational of organic and inorganic manure applications not only increase the production but also can support a stable level of the production. The mathematical model of the transformation of fertilizer in the system 'soil plant' is presented in this work. We showed that there is a stable mode for the system 'soil plants'. The strategy of the fertilizer application, permitted to provide this regime, has been determinate.*

**Résumé** – *L'application des engrais est l'une des conditions pour obtenir des cultures résistantes. L'efficacité de la contribution des engrais, tels que le phosphore, l'azote, … dépend de nombreux facteurs à savoir: les propriétés du sol, la demande des plantes en fertilisants (phosphorescence par exemple) et aussi sur la forme et la méthode d'application de ces engrais. Plusieurs résultats montrent que l'application rationnelle des engrais organiques et inorganiques est non seulement d'augmenter la production, mais prend également en charge un niveau stable de la production. Le modèle mathématique de la transformation d'engrais dans le système 'sol-plante' est présenté dans ce travail. Nous avons montré qu'il existe un mode stable pour le système 'sol-plante'. La stratégie de l'application des engrais, permettant à fournir ce régime, a été déterminée.*

**Key words**: Modeling - Dynamic Transformation - Fertilizer - System Soil-plant - Stable mode.

## 1. INTRODUCTION

The development of information and technology in the field of the chemical products for agriculture relates to a number of majorities of science directions of modern agriculture. The level of the quantitative description of the agro-ecosystem, like an object of research, determines the scientific value, the importance of the practice and the competitiveness of the agro-industrial one.

An important element of advanced information on agro-technology is the mathematical model. Like we let us know it that, for example, phosphoric manures relate to the macro-element of the nutrition of the plants. The regular use of manures increases all shapes of the nitrogen's or phosphoric compounds.

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The group of the mineral components, such as the calcium, iron, and aluminum phosphate, is exposed to the attitude of the important changes quantitative and qualitative. The quantity of the base forms of the mineral phosphate components in the various grounds is determined by the characteristics of the processes of these grounds. The proportion of aluminum phosphate increases considerably in the various grounds because this is due to the complete solubility of phosphoric manures in water.

In semi-arid agriculture for the carbonated grounds, a long application of manures makes the influence appreciable, Guseynov (1960) and Mamdov (1962). The residual component of fertilization on the nutrition of the plant activates phosphoric salt. The share of a compound of fertilization of manures can be in contact with a structure of the organic substances, roots of the plants and texture of ground. They become accessible to the plants in a process of mineralization.

The application of the radioactive isotopes changes the design about the character of the transformation of the minerals in the grounds and the availability of their plants. For the first year of fertilization by manures, the plants use only a part of entering manure and the ground can be enriched for example by digestible phosphates.

The remainder is maintained in motionless form in the ground for an application or closer action. At the time of a contribution in excess of the components in the grounds, the nutrition of the seedlings is carried out after action of the applications of old manures. It is possible to reduce, by such grounds, or to always correct the application of the quantity of components of fertilization.

The strategy of fertilization is the principle of recommendation to obtain the development of agriculture. It is obvious that to continue to use this practice of work will have as consequence the construction of a fertile ground and increases its productivity, Wang De-ren (1998).

## 2. GENERATING THE PROBLEM

Taking into account these reasons, we want to develop a mathematical model of the dynamics of made-up fertilizing in the system soil plants. The mechanism of the transformation of this element, contained in manure, is given by the following diagram (Fig. 1).

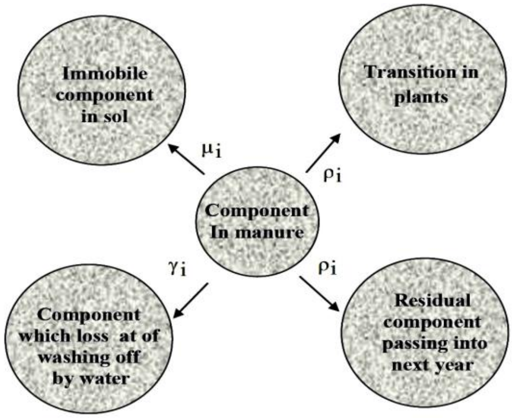


Fig. 1: Diagram of the mechanism of fertilizer transformation

The dependence of the dynamics of the absorption of made up fertilizing by the plants, $G_f$, with its quantity in the soil, $f$, is represented in the following diagram, (Fig. 2).

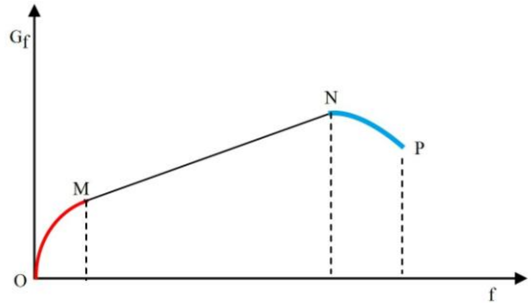


Fig. 2: Variation of the absorption of made up fertilizing by the plant

The relationship between $G_f$ and $f$ depends mainly on the plant stage of growth. Part OM corresponds to a case where the quantity of made-up fertilizing does not satisfy the minimum need for the plant. Part MN corresponds to the case where the quantity is not needed for the plant and the absorbing quantity is proportional to the quantity of fertilizer, $f$, and Part NP corresponds to the supersaturating case, when absorption of the plant decreases and accurate superabundance in the plant (poisoning effect).

From Dzhafarov (1986) we show the result of the mechanism of transition of many forms of fertilizing component for the vineyard, (Fig. 3). We will simulate the variation of the quantity of various forms of fertilizer during one year. It is known, Dzhafarov (1986), that each year roughly 35 % of phosphorus in mobile form passes in the motionless form and 15 % are required by the plant.

In the same way, not less than 20 % of phosphorus reserves in the soil are accessible to the plants. Another part of phosphorus in the soil is subjected to the difficulties of access or inaccessibility in the made up ones for the plants.

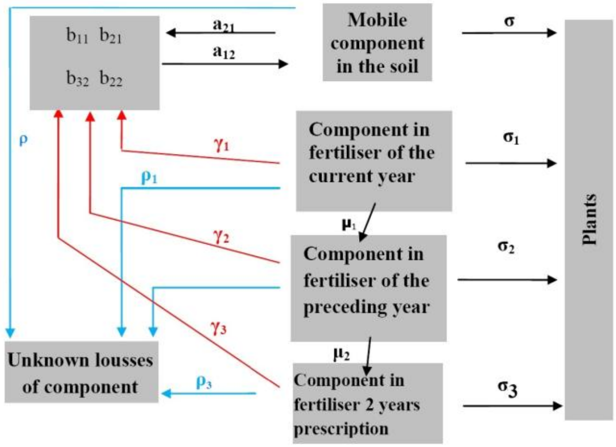


Fig. 3: the mechanism of transition from
the various forms components fertilizing for the vineyard

It is considered that the quantity of the phosphorus which was drawn up in the soil like manure remains for the 3 years a small quantity. The **Table 1** below reflects the quantitative variations of the various shapes of fertilizer.

**Table 1**: Quantity variations of the various forms of fertilizing compound

| | To mobile | To immobile | To manure | To manure | To plant | Losses |
|---|---|---|---|---|---|---|
| **Mobile in soil** | $a_{11}$ | $a_{12}$ | 0 | 0 | $\sigma$ | $\rho$ |
| **In manure of 1st year** | 0 | $\gamma_1$ | $\mu_1$ | 0 | $\sigma_1$ | $\rho_1$ |
| **In manure of 2rd year** | 0 | $\gamma_2$ | 0 | $\mu_2$ | $\sigma_2$ | $\rho_2$ |
| **In manure of 3rd year** | 0 | $\gamma_3$ | 0 | 0 | $\sigma_3$ | $\rho_3$ |
| **Immobile in soil** | $a_{12}$ | $a_{22}$ | 0 | 0 | 0 | 0 |

## 3. MODELING OF THE DYNAMIC TRANSFORMATION OF COMPONENT IN A SYSTEM SOIL PLANT

In order to determine the equations controlling the dynamic change of the products of manures in the system 'soil plants', noted by; $x_1^k$ and $x_2^k$ quantities of a component in the soil, in the mobile and motionless form respectively, at the beginning of the period of the seed (either $k = 0$).

In the same way we indicate by $x_3^k$, $x_4^k$ and $x_5^k$ the quantities of this component, which were put in the soil in the current year, that in the previous year and the year which respectively precedes the last year by fertilization.

Suitable parameters of the beginning of the next year indicated by $x_h^{k+1}$ with $j = 1$, 2, 3, 4 and 5. For the calculation of the assessment of the various forms we can write, from **Table 1**, thus:

$$\begin{cases} x_1^{k+1} = \alpha_{11}\times x_1^k + \alpha_{12}\times x_1^k \\ x_2^{k+1} = \alpha_{11}\times x_1^k + \alpha_{22}\times x_1^k + \gamma_1\times x_3^k + \gamma_2\times x_4^k + \gamma_3\times x_5^k \\ x_4^{k+1} = \mu_1\times x_3^k \\ x_5^{k+1} = \mu_2\times x_4^k \end{cases} \tag{1}$$

The total assumed compound flux which passes in the plant is calculated by

$$\phi = \sigma\times x_1^k + \sigma_1\times x_2^k + \sigma_2\times x_4^k + \sigma_2\times x_5^k \tag{2}$$

We notice that all the coefficients of the equations (1) and (2) are positive and are connected by group relations expressing the conservation laws of the total fertilizing component quantity, so we have;

$$
\begin{aligned}
&\gamma_2 + \mu_2 + \sigma_2 + \rho_2 = 1 \\
&\alpha_{11} + \alpha_{21} + \rho + \sigma = 1 \\
&\alpha_{12} + \alpha_{21} = 1 \\
&\gamma_1 + \mu_1 + \sigma_1 + \rho_1 = 1 \\
&\gamma_2 + \sigma_2 + \rho_2 = 1
\end{aligned} \tag{3}
$$

It comes to us the idea to wonder about certain questions. Is it possible each year, to use an identical quantity of manure to ensure the stabilization of the quantity of the various forms from made up fertilizing? It is possible thus to satisfy the needs for the plant in certain fertilizing compound?

### 3.1 Study of stabilization

We say that there is a stable mode, if with the annual burial of a constant quantity of manure in the soil, quantity of composed in mobile form and motionless exchange in weak and unimportant affinity. For a given quantity of the component put in the soil in the current year, $x_3^k = u$, there is a single stable mode. This translates to the system of equations:

$$
\begin{cases}
x_1^k = \alpha_{11}\times x_1^k + \alpha_{12}\times x_1^k \\
x_2^k = \alpha_{21}\times x_1^k + \alpha_{22}\times x_1^k + \gamma_1\times x_3^k + \gamma_2\times x_4^k + \gamma_3\times x_5^k \\
x_4^k = \mu_1\times u \\
x_5^k = \mu_2\times x_4^k
\end{cases} \tag{4}
$$

We replace $x_4^k$ and $x_5^k$ in equation (4) by ($\mu_1 u$) and ($\mu_1\mu_2 u$). Then the system (4) is reduced to the equivalent matrix equation:

$$
(E - A)\times\begin{pmatrix} x_1 \\ x_4 \end{pmatrix} = B \tag{5}
$$

with $B = \begin{pmatrix} 0 \\ bu \end{pmatrix}$, $A = \begin{vmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{vmatrix}$ and $E = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ is the unit matrix and $b = \gamma_1 + \gamma_2\times\mu_1 + \gamma_3\times\mu_1\times\mu_2$.

For simplicity writing we omit the superscript $k$. By taking into account the relations of conservation (3) one obtains.

$$
\det(E - A) = (\sigma + \rho)\times a_{12} > 0 \tag{6}
$$

Consequently the vector $u_\infty = \left(\lambda_1\times u + \lambda_1\times u + \mu_1\times u\times\mu_2\times u\right)$ is a fixed point of the system of equations (1), for a given value of $x_3 = u$ with: $\lambda_1 = b/\sigma$ and $\lambda_2 = (a_{12} + \sigma)/a_{12}\times\sigma$.

### 3.2 Process of stabilization

Let us choose the mode $x_3^{k+1} = x_3^k = u$. For the 3rd year the dynamics of variation of made up fertilizer will be described by the system of equations according to:

$$\begin{cases} x_1^{k+1} = a_{11}\times x_1^k + a_{12}\times x_1^k \\ x_2^{k+1} = a_{21}\times x_1^k + a_{22}\times x_1^k + \gamma_1\times u + \gamma_2\times u + \gamma_3\times x_5^k \\ x_4^{k+1} = \mu_1\times u \\ x_5^{k+1} = \mu_2\times x_4^k \end{cases} \tag{7}$$

By replacing $x_4^k$ and $x_5^k$ in the second equation of this system, we can write it as follows:

$$x^{k+1} = A\times x^k + B \tag{8}$$

where $k = 3,4,5,\ldots,$ $B = (0,b)$ and $b = \gamma_1 + \gamma_2\times\mu_1 + \gamma_3\times\mu_1\times\mu$

**3.2.1 Proposal 1**

That is to say $u_\infty = \left(\lambda_1\times u + \lambda_2\times u\right)$ for a given value, $u$, if the condition

$$a_{21} < a_{12} < a_{21} + \sigma \tag{9}$$

is checked, then $\lim\limits_{k\to\infty}(x)^k = u_\infty$

To justify it, let us consider

$$x^{k+1} - x^k = ((A(x))^k - x^{k-1})$$

From this one has

$$\|A\|\times\left\|x^k - x^{k-1}\right\| = \left\|x^{k+1} - x^k\right\|$$

where

$$\|A\| = \max[a_{11} + a_{11}, a_{21} + a_{22}]$$

According to the condition (10)

$$a_{11} + a_{12} < a_{11} + a_{21} + \sigma < 1$$

$$a_{21} + a_{22} < a_{12} + a_{22} < 1$$

Consequently, the matrix $A$ is a contracted form and $\{u^k\}_{(k=3,4,5,\ )}$ is like a fundamental sequence:

$$u_\infty = A\times u_\infty + b\times u$$

Then

$$x^{k+1} - u_\infty = (A\times x^k + b\times u) - (A\times u_\infty + b\times u) = A(x^k - u_\infty) \to \beta$$

with $\beta = (0, 0)$

Then the proposal is justified. In addition, the soil-plant system satisfying the condition (9) is termed favourable.

### 3.2.2 Proposal 2

For a certain reserve of compound fertilizer in a favourable soil, it is possible to make such a stabilizing element, which for the third year, would meet the needs of plants in phosphorus.

To proof this, note $\phi_0$, the total need of the plant in this compound, is known. Let's write the equation of the assessment

$$\phi_0 = \sigma\times\lambda_1\times u + \sigma_1\times u + \sigma_2\times u\times\mu_1 + \sigma_3\times u\times\mu_2 \tag{10}$$

From this, we can write:

$$u = \frac{\phi_0}{(\sigma\times\lambda_1 + \sigma_1 + \sigma_2\times\mu_1 + \sigma_3\times\mu_2)} \tag{11}$$

Then such choice (preceding proposal) will result in stabilization and simultaneously will satisfy the need for the plant. Let us suppose that at the beginning of the k-th year the quantities of the various forms of phosphorus ($x_1^k$, $x_2^k$, $x_3^k$, $x_5^k$) are known. Whether are possible operating by the quantity of phosphorus, apply in to soil, $x_3^k = u_1$, $x_3^{k+1} = u_2$, $x_3^{k+2} = u_3$, $x_3^{k+3} = u_4$, during sequence 4 years to achieve a stable mode, i.e., that on ending 4 years, quantity of phosphorus, accordingly became: $x_1^{k+4} = \lambda_1\times u$, $x_2^{k+4} = \lambda_2\times u$, $x_4^{k+4} = \mu_1\times u$, $x_5^k = \mu_2\times u$, for the beforehand chosen value $u$. In the case, $\{u^k\}_{k=1,2,3,4}$ will be called an application strategy of fertilizing compound.

### 3.2.3 Proposal 3

For a any initial set (11) and final expectations ($x_1^k$, $x_2^k$, $x_4^k$, $x_5^k$) exist the strategy to apply a fertilizing compound, if the following inequality is true.

$$\begin{gathered} a_{11}\times(a_{11}\times\gamma_1 + a_{22}\times\gamma_1 + \mu_1\times\gamma_2)\times(a_{22} - a_{11} \pm \sqrt{(a_{11}+a_{22}) - 4(a_{11}+a_{22}+\sigma+\rho)} \\ \neq 2(a_{12}(\gamma_1\times a_{12}\times a_{21} + a_{22}(\gamma_1\times a_{22}+\gamma_2\times\mu_1)) + \gamma_1\times\mu_2) \end{gathered} \tag{12}$$

Let us proof this proposal. For doing this, we note that

$$x^k = \begin{vmatrix} x_1^k \\ x_2^k \\ x_4^k \\ x_5^k \end{vmatrix} \qquad B = \begin{vmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{22} & a_{21} & \gamma_2 & \gamma_3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} \qquad b = \begin{vmatrix} 0 \\ \gamma_1 \\ \mu_1 \\ 0 \end{vmatrix}$$

Then the assessment of composed for the fourth year end must be described by the following sequence:

$$x^{k+12} = B\times x^k + B\times bu^k$$

$$x^{k+2} = B^2\times x^k + B\times bu^k + bu^k$$

$$x^{k+3} = B^3\times x^k + B^2\times bu^k + B\times bu^k + bu^{k+2}$$

$$x^{k+4} = B^4 \times x^k + B^3 \times bu^k + B^2 \times bu^k + B \times bu^{k+2} + bu^{k+3}$$

The last equation can be written in the equivalent form

$$x^{k+4} - B^4 \times x^k = B^3 \times bu^k + B^2 \times bu^k + B \times bu^{k+2} + bu^{k+3}$$

And so on.

The last equation can be written as

$$x^{k+4} - B^4 \times x^k = \begin{vmatrix} b_{31} & b_{21} & b_{11} & 0 \\ b_{32} & b_{22} & b_{12} & b_{02} \\ 0 & 0 & 0 & b_{03} \\ 0 & 0 & b_{14} & 0 \end{vmatrix} \times \begin{vmatrix} u^k \\ u^{k+1} \\ u^{k+2} \\ u^{k+3} \end{vmatrix} \qquad (13)$$

The system (14) is solved if its matrix is not degenerated. So let us compute the determinant of its matrix;

$$\det \begin{vmatrix} b_{31} & b_{21} & b_{11} & 0 \\ b_{32} & b_{22} & b_{12} & b_{02} \\ 0 & 0 & 0 & b_{03} \\ 0 & 0 & b_{14} & 0 \end{vmatrix} = b_{03} b_{14} \det \begin{vmatrix} b_{31} & b_{21} \\ b_{32} & b_{22} \end{vmatrix} = \mu_1^2 \mu_1 \det \begin{vmatrix} b_{31} & b_{21} \\ b_{32} & b_{22} \end{vmatrix}$$

By the form, the columns of determinant are linked by the evident relations

$$\begin{vmatrix} b_{31} \\ b_{32} \end{vmatrix} = A \begin{vmatrix} b_{21} \\ b_{22} \end{vmatrix}$$

Consequently

$$\det \begin{vmatrix} b_{31} & b_{21} \\ b_{32} & b_{22} \end{vmatrix}$$

And if $\vec{V} = \begin{vmatrix} b_{21} \\ b_{22} \end{vmatrix}$ is not an appropriate eigenvector of the matrix A to one of its eigenvalues, which is equal to

$$\nu_{1,2} = \left( a_{11} \times a_{22} \pm \sqrt{(a_{11} + a_{22}) - 4((a_{11} + a_{22} + (\sigma + \rho) a_{11} - 1)/2)} \right)$$

i.e if the previous inequality is true, we have:

$$a_{11} \times a_{12} (a_{11} \times \gamma_1 + a_{22} \times \gamma_1 + \mu_1 \times \gamma_2) + a_{21} \times (\gamma_1 \times a_{12} \times a_{22} \times (a_{22} \times \gamma_1 + \mu_1 \times \gamma_2))$$
$$+ \gamma_3 \times \mu \neq \nu_{1,2} \times a_{12} \times (a_{11} \times \gamma_1 + a_{22} \times \gamma_1 + \mu_1 \times \gamma_2)$$

Taking into account an explicit mode eigenvalues $\nu_{1,2}$, we obtain the condition (12).

## 4. CONCLUSION

We have established that the annual regulation dose of applied fertilizer is most effective approach for a given soil plant system, which is characterized by a whole complex of parameters when calculated. Also we have shown that it is a stable regime for a system soil plant.

A strategy has been determinate to obtain this stable regime. Then, according to the initial parameters, it is possible to work directly under the regime of the application of a constant quantity of the component in the soil or to establish this regime along the first four years.

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