

Numerical study of the transient heat and moisture transfer through the single and double walls

Youcef Tamene ^{1*}, Said Abboudi ² and Cherif Bougriou ³

¹ Department of Mechanical Engineering, University Kasdi Merbah, Ouargla, Algeria

² Systems and Transport Laboratory, Department of Mechanical Engineering and Design, University of Technology, Belfort Montbéliard, Site Sévenans, 90010 Belfort Cedex, France

³ Laboratory of Industrial Energy Systems Studies,
Department of Mechanics, University of Batna, Algeria

(reçu le 10 Janvier 2013 – accepté le 30 Septembre 2013)

Abstract - *In this work, a numerical study of a coupled transient heat and moisture transfer was realized. The program developed is simple compared with those developed by other researchers in the literature. We have exploited it, to compare the evolution of temperature and the moisture content for simple and double layers walls of the same type and the same thickness in order to resolve the interface problem, then the influence of some boundary conditions was explored in order to see their impact on temperature and moisture evolution through two layer materials.*

Résumé - *Dans ce travail, une étude numérique sur la chaleur transitoire couplée et sur le transfert d'humidité a été réalisée. Le programme développé est simple par rapport à ceux développés par d'autres chercheurs dans la littérature. Nous l'avons exploité, pour comparer l'évolution de la température et la teneur en humidité sur des couches de simples et doubles parois, du même type et de même épaisseur afin de résoudre le problème d'interface, puis l'influence de certaines conditions aux limites a été étudiée afin de voir son impact sur la température et l'évolution de l'humidité à travers les deux couches de matériaux.*

Key words: Heat transfer - Moisture content - Unsteady – Coupled - Two layers - Numerical.

1. INTRODUCTION

In building area, comprehension of the heat and moisture transfer is important to ensure thermal comfort and economize energy, in addition, moisture causes damage to building materials and has an impact on occupants' health.

Most of research is carried out by using phenomenological macroscopic models, one of the accepted macroscopic models for studying heat and moisture transfer through porous media is the Philip *et al.* [1], which uses as driving potentials the temperature and moisture content gradients. Crausse *et al.* [2] presented two models for determining the temperature and moisture distribution within a wall, one of this models take into account the hysteresis phenomena.

Mendes *et al.* [3] have shown the effects of moisture on sensible and latent conduction loads by using a simple heat and mass transfer model with variable material properties, under varying boundary conditions. Mendes *et al.* [4] presents a mathematical formulation applied to numerically solver, showing that moisture content

* tamene.youcef@univ-ouargla.dz or tamene_y@yahoo.fr

gradients can be used as driving forces for heat and moisture transport calculation through the interface between porous materials with different pore size distribution functions.

Tamene *et al.* [5] proposed a numerical study of the heat and mass transfer through a wall submitted to moisture and heat convective exchange with the ambient. The temperature effect on the moisture diffusion and vice versa is presented for two usual materials in the presence of solar heat flux.

Experimentally, Talukdar *et al.* [6] conducted a comparison between a numerical code and a simplified method for the experimental study of mass and heat transfer for two types of insulation (cellulose, plywood) used in buildings.

Olalekan [7] studied in his thesis the Characteristics of Spruce Plywood, under the influence of heat and moisture transfer. A dynamic model for evaluating the transient thermal and moisture transfer behaviour in porous building materials was presented by Qin *et al.* [8], where heat and moisture transfer are simultaneously considered, and their interactions are studied.

Garbalinska *et al.* [9] provided a description of moisture transport in building materials by application of a non-linear diffusion model, in which the effective coefficient of diffusion, being a function of moisture content. They propose a method of determination of this coefficient through correlation of the theoretical desorption isotherms with the experimental ones. Nguyen *et al.* [10] developed a physically based model describing the coupled ion and moisture transport by combining theories of liquid water and water vapour transport with aqueous electrolyte theory.

They derive the set of governing differential equations describing simultaneous movement of water in the vapour and liquid phases and consequent transport of ions in unsaturated porous media. The equations are developed in one-dimension, assuming isothermal conditions. A computer program has been developed to solve this problem.

Dos Santos *et al.* [11] proposes a two-dimensional mathematical model considering the coupled heat, air and moisture transport through unsaturated building hollow bricks. Simulations for evaluating hydrothermal performance were performed for massive, hollow and insulating bricks.

In the present work, we study a coupled heat and moisture transfer through a double layer wall submitted to moisture and heat convective exchange with the ambient. The heat and mass balance equations, in the transient regime, was performed by finite differences method according to the Cranck-Nicolson scheme, a program in Fortran was realized and compared with a single layer in order to resolve the interface problem.

The results obtained by our program were validated with those obtained experimentally. We analyzed the influence of the boundary conditions on the evolution of temperature and the moisture through the wall and on the interaction between them.

2. THE FORMULATION OF THE PROBLEM

The studied material is a plywood plate of four centimeters thick. The different physical properties used in these studies are obtained experimentally by Talukdar *et al.* [6] and Olalekan [7].

The governing equations are given by {Eqs. (1) and (2)}; they were derived from conservation of mass and energy flow in one dimensional elemental volume since, the temperature and the moisture content gradients, are predominantly in the x direction.

Therefore, the energy conservation equation is written as:

$$\rho \times C_p \times \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(\lambda \times \frac{\partial T}{\partial x} \right) + \rho_L \times L_V \times \frac{\partial}{\partial x} \left(D_{T_V} \times \frac{\partial T}{\partial x} \right) + \rho_L \times L_V \times \frac{\partial}{\partial x} \left(D_{\theta_V} \times \frac{\partial \theta}{\partial x} \right) \quad (1)$$

and the mass conservation equation is

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left(D_T \times \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial x} \left(D_\theta \times \frac{\partial \theta}{\partial x} \right) \quad (2)$$

where, $D_T = D_{T_L} + D_{T_V}$ and $D_\theta = D_{\theta_L} + D_{\theta_V}$.

By supposing the following hypotheses:

- Thermal conductivity and diffusivity constants,
- The different coefficients are constant.
- The interface contact is perfect.

We obtained the following form of { Eqs (1) and (2)}:

$$\frac{\partial T}{\partial t} = \lambda' \times \frac{\partial^2 T}{\partial x^2} + S_\theta \quad (3)$$

$$\frac{\partial \theta}{\partial t} = D_\theta \times \frac{\partial^2 \theta}{\partial x^2} + S_T \quad (4)$$

Where,

$$\lambda' = \left(\frac{\lambda + \rho_L \times L_V \times D_{T_V}}{\rho \times C_p} \right) \quad D'_{\theta_V} = \frac{\rho_L \times L_V \times D_{\theta_V}}{\rho \times C_p} \quad S_\theta = D'_{\theta_V} \times \frac{\partial^2 \theta}{\partial x^2} \quad S_T = D_V \times \frac{\partial^2 T}{\partial x^2}$$

The associated boundary conditions are illustrated on figure 1.

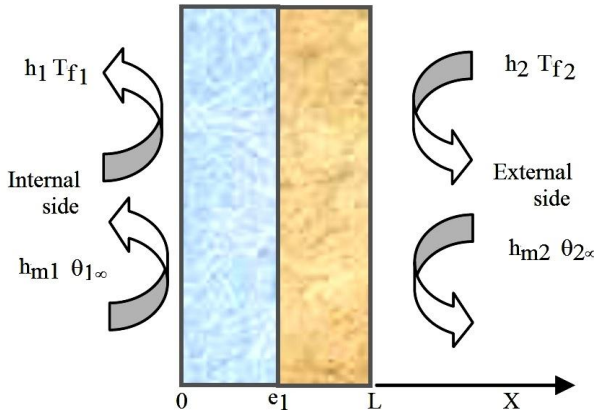


Fig. 1: Physical model

From mass transfer-

$$-\rho_L \times D_T \times \frac{\partial T}{\partial x} - \rho_L \times D_\theta \times \frac{\partial \theta}{\partial x} = h_{m1} \times (\rho_1 - \rho_{1\infty}) \quad \text{at } x = 0 \quad (5)$$

$$\begin{aligned} -(\rho_L \times D_T)_1 \times \frac{\partial T_1}{\partial x} - (\rho_L \times D_\theta)_1 \times \frac{\partial \theta_1}{\partial x} = \\ -(\rho_L \times D_T)_2 \times \frac{\partial T_2}{\partial x} - (\rho_L \times D_\theta)_2 \times \frac{\partial \theta_2}{\partial x} \quad \text{and } \theta_1 = \theta_2 \quad \text{at } x = e_1 \end{aligned} \quad (6)$$

$$-\rho_L \times D_T \times \frac{\partial T}{\partial x} - \rho_L \times D_\theta \times \frac{\partial \theta}{\partial x} = h_{m1} \times (\rho_{N\infty} - \rho_2) \quad \text{at } x = L \quad (7)$$

For heat transfer-

$$-\lambda \frac{\partial T}{\partial x} - \rho_L \times L_V \times D_{T_V} \times \frac{\partial T}{\partial x} - \rho_L \times L_V \times D_{\theta_V} \times \frac{\partial \theta}{\partial x} = h_1 \times (T_{f1} - T(0)) \quad \text{at } x = 0 \quad (8)$$

$$-\lambda_1'' \times \frac{\partial T_1}{\partial x} - D_{\theta_{V1}}'' \times \frac{\partial \theta_1}{\partial x} = -\lambda_2'' \times \frac{\partial T_2}{\partial x} - D_{\theta_{V2}}'' \times \frac{\partial \theta_2}{\partial x} \quad T_1 = T_2 \quad \text{at } x = e_1 \quad (9)$$

$$-\lambda \frac{\partial T}{\partial x} - \rho_L \times L_V \times D_{T_V} \times \frac{\partial T}{\partial x} - \rho_L \times L_V \times D_{\theta_V} \times \frac{\partial \theta}{\partial x} = h_2 \times (T - T_{f2}) \quad \text{at } x = L \quad (10)$$

With, $\lambda'' = \lambda + \rho_L \times L_V \times D_{T_V}$ and $D_{\theta_V}'' = \rho_L \times L_V \times D_{H_V}$.

The mass convection coefficients h_{m1} , h_{m2} are related respectively, to the heat transfer coefficients h_1 , h_2 by the Lewis relation.

Initial conditions-

$$T = T_0 = 20^\circ\text{C}, \quad \theta = \theta_0 = 0.2 \quad \text{for } t = 0 \quad \text{and } 0 \leq x \leq L \quad (11)$$

The mass densities ρ_1 , $\rho_{1\infty}$, ρ_2 , $\rho_{2\infty}$ are computed by the following equation:

$$\rho = \frac{p}{R_h \times T}, \quad \text{with } R_h = \frac{R_S}{1 - \left(\theta \frac{p_{\text{sat}}}{p} \right) \times \left(1 - \frac{R_S}{R_V} \right)}$$

$R_S = 287.06 \text{ J/kg.K}$ universal gas constant of dry air.

$R_V = 461 \text{ J/kg.K}$ universal gas constant of water vapour.

And the saturation vapour pressure p_{sat} is determined by Magnus formula [12]

$$p_{\text{sat}} = 611.213 \exp\left(\frac{17.5043 T}{241.2 + T}\right) \quad (12)$$

Finally we obtained:

$$\rho(\theta, T, P) = \frac{1}{A} \times \left(P - B \times \theta \times \exp\left(\frac{C \times T}{D + T}\right) \right) \quad (13)$$

Where, $A = 287.06 \times (T + 273.15)$, $B = 230.617$, $C = 17.5043$ and $D = 241.2$.

3. NUMERICAL RESOLUTION

The resolution of the coupled system of equations (1) and (2) with boundary conditions (4), (5), (6) and (7) and initial conditions (8) is performed by finite differences method according to the Cranck-Nicolson scheme, [13].

At each time, the Tri Diagonal Matrix Algorithm of Thomas (TDMA) is used to solve the obtained algebraic systems of temperature and humidity.

4. RESULTS

In order to analyze the interface conditions, we propose a comparison between a simple and a double layer wall. The two walls are in plywood and have the same thickness. For the double layer, at the interface, two coefficients are introduced numerically H_{T_i} and H_{θ_i} , in order to assure the continuity of heat and mass fluxes, temperatures and moisture. After discretization of the balance equations, these coefficients appear in the following forms:

$$F_{T_i} = \frac{2 \times H_{T_i} \times \Delta x}{\rho_l + L_v \times D_{T_v}}, \quad F_{\theta_i} = \frac{2 \times H_{H_i} \times \Delta x}{\rho_l \times D_{H_v}}$$

They depend on the grid and physical properties of layers. Several trials were realized with different value of H_{T_i} , H_{θ_i} , F_{T_i} , F_{θ_i} , Fig. 2 and Fig. 3.

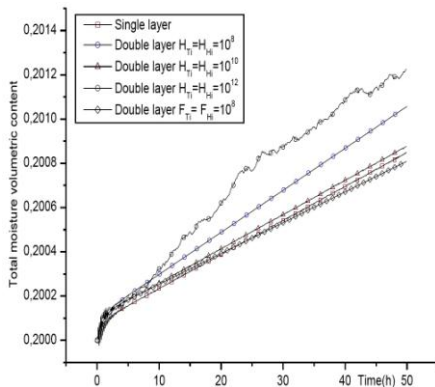


Fig. 2: Evolution of the internal side moisture content

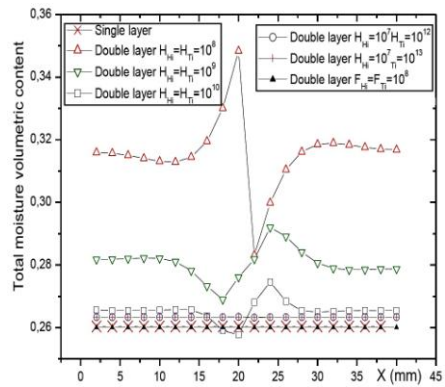


Fig. 3: Moisture content profiles after 50 hours

The good choice, for having a good agreement with results obtained for a simple layer, is using the values of F_{T_i} and F_{θ_i} equal to 10^8 . For the remainder of this study, we use this value for both coefficients.

The temperature profile through a double layer and single layer walls after 50 hours was presented in Fig. 4. In both cases, we take the same materials and thickness. We see a good agreement.

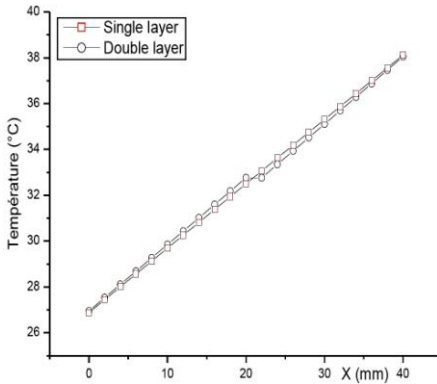


Fig. 4: Temperature profiles after 50 hours

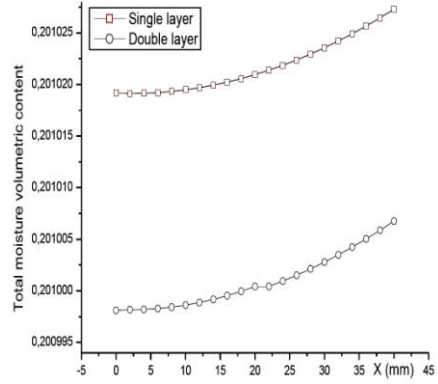


Fig. 5: Moisture content profiles after 50 hours

For the moisture content, (Fig. 5), we note a slight difference between a double and a simple wall of the order of 0.0003. We also note that in the interface we have the same values at 22 mm and 22.25 mm for temperature and moisture content. This can be explained by the fact that we have supposed equality of temperature and flux at the interface.

In Fig. 6, we note that the temperature evolution is same in the two cases. The steady state is attained after 5 hours of exposure, with external, interface, and internal temperatures, respectively equal at 38 °C, 32 °C and 27 °C.

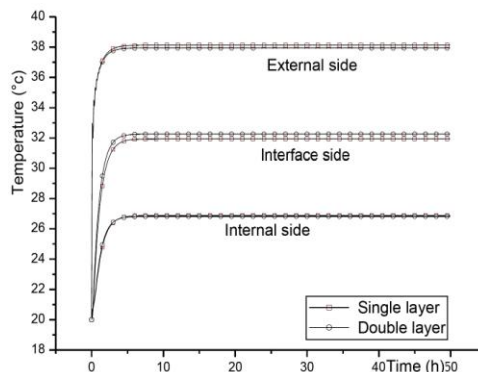


Fig. 6: The temperature evolution

Afterwards, we have analyzed the influence of the boundary conditions (outdoor temperature and convective exchange coefficient) on two layers wall behaviour. The first is composed of two identical layers of plywood, and the second is composed of two different layers (plywood and mortar).

4.1 Influence of the convective exchange coefficient- double layer (plywood)

In this section, we fixed the outdoor and indoor temperature respectively at 35 °C and 22°C, and the outdoor and indoor relative humidity respectively at 70 % and 50 %.

The convective exchange coefficient at the external side is taken equal at 13 W/m².°C. The simulation time is 50 hours. We present in Fig. 7, the temperature profile for four different convective exchange coefficients ($h_1 = 0, 3, 5$ and $7 \text{ W/m}^2\text{.}^\circ\text{C}$).

We note that the internal side temperature decreases faster than the external side temperature when h_1 increase.

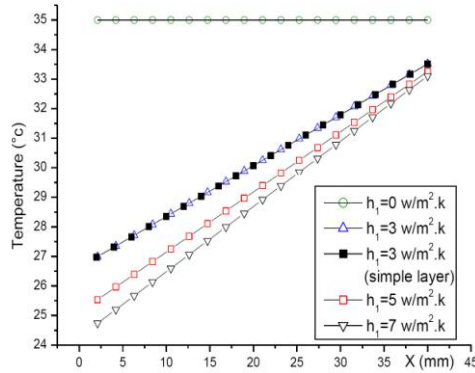


Fig. 7: Temperature through a double layer wall for different values of h_1

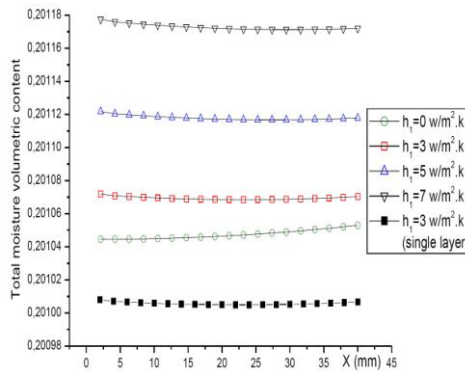


Fig. 8: Moisture content through a double layer wall for different values of h_1

For the moisture content, Fig. 8, we observe the inverse, when the convective exchange coefficient h_1 increase, it decrease. The temperature and moisture content for a single layer wall has been presented in the same figure, for $h_1 = 3 \text{ W/m}^2\text{.}^\circ\text{C}$ to ensure results.

The temperatures evolution, Fig. 9 shows a diminution in response time with augmentation of the convective exchange coefficient h_1 .

For example, it is 8 hours for $h_1 = 0 \text{ W/m}^2\text{.}^\circ\text{C}$, and 5 hours for $h_1 = 7 \text{ W/m}^2\text{.}^\circ\text{C}$.

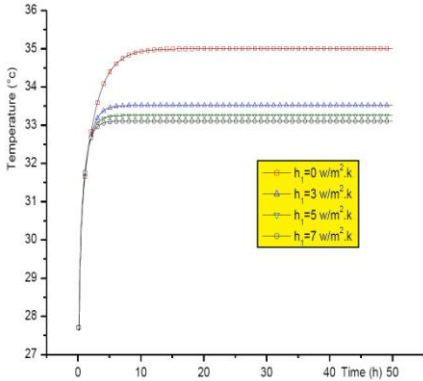


Fig. 9: Temperature of the external side for different values of h_1

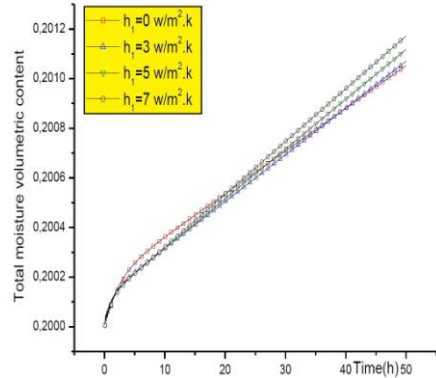


Fig. 10: Moisture content of the external side for different values of h_1

In Fig. 10, we note that during the initial 20 hours the moisture content, increase linearly in the same way with time, for different h , only for $h_1 = 0$, but after 20 hours it increases more rapidly in function of h . This is mainly due to the fact that the temperature diffuses faster than the moisture.

4.2 Influence of outdoor temperature- double layer (plywood)

Figure 11 and 12 shows the influence of the outdoor temperature T_{f2} on the evolution of moisture content through a double layer wall. The internal side is supposed adiabatic. We note that it increase linearly with time except when the outdoor temperature is lower than the initial temperature, where the moisture content decreases at the beginning of the transient regime, time that the moisture diffuses in the wall.

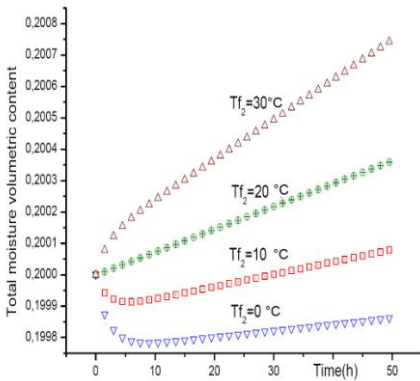


Fig. 11: Moisture content of the internal side

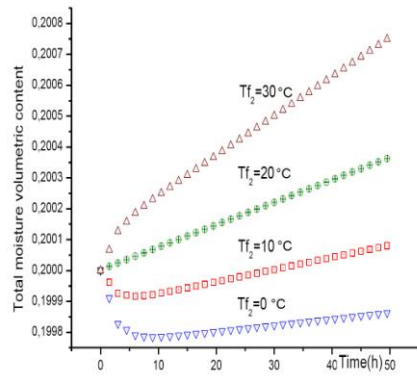


Fig. 12: Moisture content of the external side

4.3 Influence of the convective exchange coefficient- double layer (plywood-mortar)

In this section, we choose a wall composed by two layers with the same thickness (2 cm); the internal layer is in plywood; the external in mortar. The boundary and analyzed

conditions are identical to those of the previous case. In Fig. 13, we note that the time required to reach steady state is longer than the previous case (50 hours). When there is a convective heat transfer, this time is less, but it decreases as h_1 increases. For moisture content, Fig. 14, we observed the same allure as the previous case. It increases more rapidly for mortar than for plywood.

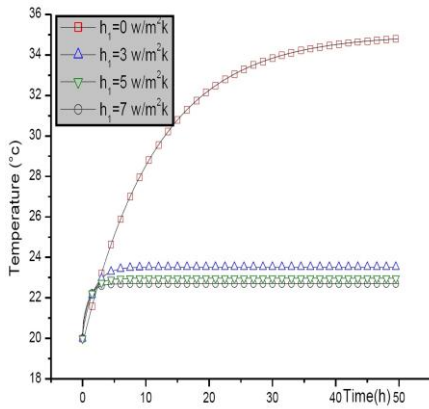


Fig. 13: Temperature of the external side for different h_1

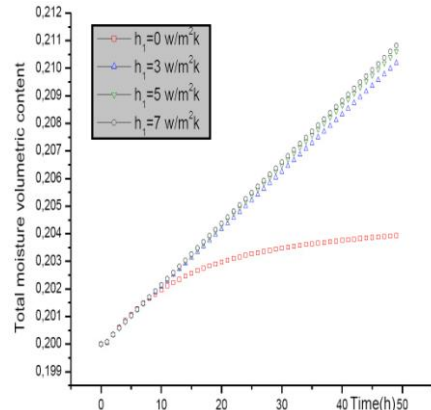


Fig. 14: Moisture content of the external side for different h_1

4.4 Influence of the outdoor temperature- double layer (plywood-mortar)

The temperature and moisture content profile after 50 hours are presented respectively, for two different outdoor temperatures, in Fig. 15, we note that when the outdoor temperature is inferior to the initial temperature (20 °C).

The moisture content decrease relative to the initial (0.2), (Fig. 16), while the temperature increase in the presence of the moisture.

The opposite is observed when the outdoor temperature is equal to the initial temperature.

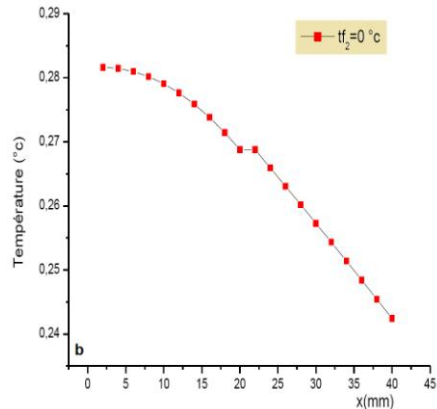
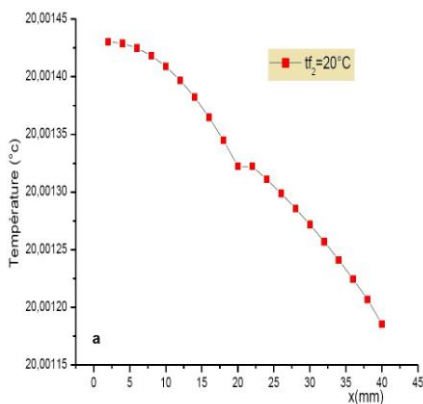


Fig. 15: Temperature profile after 50 hours

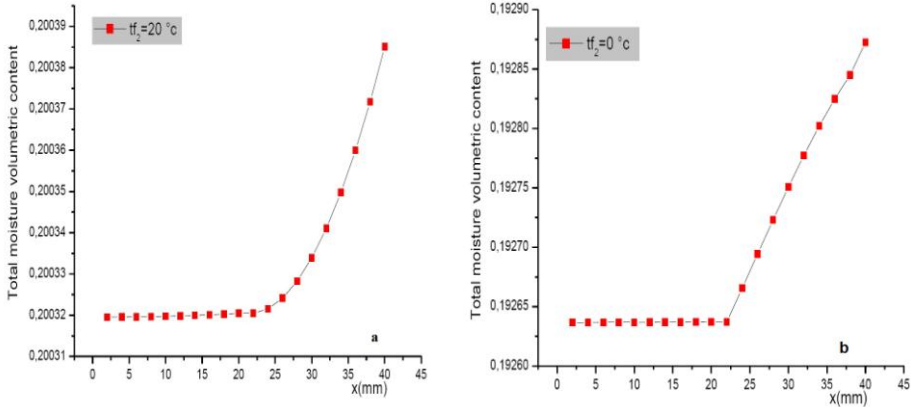


Fig. 16: Moisture content profile after 50 hours

5. CONCLUSION

The numerical resolution, of heat and mass balance equations, in the transient regime, was firstly performed for the single wall case. The developed program was validated with experimentally results. Then it was exploited for study a double layer wall.

The interface problem between the layers was analyzed, by comparing simple and double layer walls of the same type and the same thickness. A numerical solution was found for the interface problem; it permitted us to improve the accuracies of the calculation, by this fact, the program became unconditionally stable.

The influence of the convective exchange coefficients and the ambient temperature variation, on thermal and mass behaviour, was analyzed, in case of the double layer wall, with same and different physical properties.

We have noted that the internal side temperature decreases faster than the external side temperature when h_1 increases. For the moisture content, we have observed the inverse, when the convective exchange coefficient h_1 increases, it decreases. When the outdoor temperature is inferior to the initial temperature, the moisture content decreases relative to the initial moisture content, while the temperature increases in the presence of moisture.

The opposite is observed when the outdoor temperature is equal to the initial temperature. The program developed allows us to study the transfer of coupled heat and moisture through building materials, regardless of the values of the boundary conditions, which will allow us to characterize some local building materials.

NOMENCLATURE

D_T : Mass transport coefficient associated to a temperature gradient, ($\text{m}^2/\text{s.K}$)

D_{T_V} : Vapour phase transport coefficient associated to a temperature gradient, ($\text{m}^2/\text{s.K}$)

D_θ : Mass transport coefficient associated to a moisture content gradient, (m^2/s)

D_{θ_V} : Vapour phase transport coefficient associated to a moisture content gradient, (m^2/s)

C_p : Specific heat, (J/kg.K)	L : Wall thickness, (m)
h : Heat convection transfer coefficient, (W/m ² .K)	h_m : Mass convection transfer coefficient, (m/s)
L_V : Heat of vaporization, (J/kg)	N : Node number
P : Pressure, (Pa)	t : Time, (s)
T : Temperature, (°C)	T_f : Air Temperature, (°C)
λ : Thermal conductivity, (W/m.K)	ρ : Mass density, (kg/m ³)
θ : Total moisture volumetric content, (m ³ of water/m ³ of porous material)	
L- Liquid ; V- Vapour ; 0- Initial	1- Internal side ; 2- External side
∞ - Far wall	

REFERENCES

- [1] J.R. Philip and D.A. De Vries, 'Moisture Movement in Porous Materials under Temperature Gradient', Transactions of American Geophysical Union, Vol. 38, pp. 222 – 232, 1957.
- [2] P. Crausse, J.P. Laurent et B. Perrin, 'Influence des Phénomènes d'Hystérésis sur les Propriétés Hydriques des Matériaux Poreux', Revue Générale de Thermique, Vol. 35, pp. 95 – 106, 1996.
- [3] N. Mendes, F.C. Winkelmann, P.C. Philipi and R. Lamberts, 'Moisture Effects on Conduction Loads', Energy and Building, Vol. 35, N°7, pp. 631 - 664, 2003.
- [4] N. Mendes and P.C. Philipi, 'A Method for Predicting Heat and Mass Transfer Through Multilayered Walls Based on Temperature and Moisture Content Gradients', International Journal of Heat and Mass Transfer, Vol. 48, N°1, pp. 37 - 51, 2005.
- [5] Y. Tamene, S. Abboudi and C. Bougriou, 'Study of Heat and Moisture Diffusion Through a Wall Exposed To Solar Heat Flux', Journal of Engineering Science and Technology, Vol. 6, N°4, pp. 429 – 444, 2011.
- [6] P.T. Talukdar, S.O. Olutmayin, O.F. Osanyintola and C.J. Simonson, 'An Experimental Data Set For Benchmarking 1D, Transient Heat and Moisture Transfer Models of Hygroscopic Building Materials. Part II: Experimental, Numerical and Analytical Data', International Journal of Heat and Mass transfer, Vol. 50, N°23-24, pp.4527 – 4539, 2007.
- [7] F.O. Olalekan, 'Transient Moisture Characteristics of Spruce Plywood', Master Thesis, University of Saskatchewan, Canada, 2005.
- [8] M. Qin, R. Belarbi, A. Ait-Mokhtar and A. Seigneurin, 'An Analytical Method to Calculate the Coupled Heat and Moisture Transfer in Building Materials', International Communications in Heat and Mass Transfer, Vol. 33, N°1, pp. 39 – 48. 2006.
- [9] H. Garbalinska, S.J. Kowalski and M. Staszak, 'Moisture Transfer between Unsaturated Cement Mortar and Ambient Air', Transport in Porous Media, Vol. 85, N°1, pp. 79 – 96, 2010.
- [10] T.Q. Nguyen, J. Petkovic, P. Dangla and V.B. Bouny, 'Modelling of Coupled Ion and Moisture Transport in Porous Building Materials', Construction and Building Materials, Vol. 22, N°11, pp. 2185 – 2195, 2008.
- [11] G.H. Dos Santos and N. Mendes, 'Heat, Air and Moisture Transfer Through Hollow Porous Blocks', International Journal of Heat and Mass Transfer, Vol. 52, N°9-10, pp. 2390 – 2398, 2009.

- [12] Guide to Meteorological Instruments and Methods of Observation, annex 4-B, *Formulae for the Computation of Measures of Humidity*, World Meteorological Organization, 2008.
- [13] F.G. Curtis, 'Applied Numerical Analysis', Addison Wesley Publishing Company, 1978.