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A Dynamic Mathematical Model to Predict the Performance of Passive Cooling System by Evapo-Reflective Roof for Hot Dry Climates

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Résumé - *Un modèle mathématique dynamique a été développé pour connaître l'efficacité d'un système de refroidissement passif utilisant une toiture réflective-évaporative. Le système est composé d'une dalle en béton sur laquelle sont déposés des galets insérés dans une mare d'eau contenue dans une enceinte étanche qui empêche toute perte d'eau. La face supérieure de l'enceinte est recouverte d'une tôle en aluminium peinte en blanc pour augmenter au maximum la réflexion de radiation pendant le jour. Le système est implanté dans la ville de Laghouat au sud, centre de l'Algérie (latitude +33.46N et longitude +2.56 E). L'écart de température remarqué entre le jour et la nuit, en été dans cette région, permet de stocker les frigories la nuit et de les utiliser pendant la journée. Le soir, la température de la plaque d'aluminium est inférieure à la température des galets insérés dans l'eau. La vapeur à l'intérieur du toit se condense et par conséquent tombe par gravité. Cet effet de la pipe de chaleur canalise la chaleur à l'extérieur. L'échange de chaleur est amélioré par radiation entre les deux surfaces internes humides. L'efficacité de ce système est étudiée en utilisant la méthode des différences finies. Les calculs numériques exécutés pour diverses températures externes et radiations solaires ont montré que le système proposé a produit le refroidissement d'une façon significative. Comme résultat de ceci, la température maximale de l'air dans la pièce peut être gardée quelques degrés au-dessus de la température minimale de l'air extérieur pendant le jour. L'association au système de la ventilation naturelle nocturne de vingt heures jusqu'à sept heures peut baisser la température intérieure de 3 à 4 degrés. Le travail continue.*

Abstract - *A dynamic mathematical model used to predict the performance of passive cooling system by evapo-reflective roof in buildings for hot arid climates has been developed. The cooling system consists of a roof composed of concrete ceiling and flat aluminium plate, separated with air space partially filled with rocks and small quantity of water. Low emissive aluminium sheet with white painting on the upper surface, reduces the heat transfer by radiation during the daytime, an external treatment of the roof used to provide mass transfer to the outside (water vapour). At night, the temperature of the aluminium sheet falls below the temperature of the rocks water upper surface. Water vapour inside the roof condensates and falls by gravity. This heat pipe effect carries heat outwards. Heat exchange is improved by radiation between two humid internal surfaces. The efficiency of this cooling system is studied using finite differences method. Numerical calculations performed for different external temperatures and solar radiation show that the cooling produced by such system is significant. As a result of this, the mean air temperature in the room may be kept a few degrees over the minimum outdoor temperature throughout the day. Allowing natural ventilation of the building in the evening could further, lower the indoor air temperature by 3 to 4°C. The work is continuing.*

Keywords: Evaporative cooling - Evapo-reflective roof - Hot dry climate - Night ventilation - Dynamic model.

1. INTRODUCTION

In hot climates such as southern Algeria, excessive heat is the major problem that causes human thermal discomfort. Space cooling is therefore the most desirable factor for the inhabitants. Various examples of dwellings responsive to climatic constraints were found in vernacular architecture throughout the world. Compact cellular layout with minimum external surface exposure to the sun, whitewash surfaces to reduce absorptivity, blind external facades, courtyards, vegetation to provide humidity and shade, heavy buildings with high thermal capacity materials are common passive features in most hot arid climates such M'Zab settlements in southern Algeria, Egypt and Iran [1-4]. Wind towers for cooling ventilation are well known in Iranian and Middle East architecture, which along with cooling of the air by water evaporation kept the building comfortable in hot periods [5]. Building underground to take the advantage of large thermal storage capacity of the earth. They are used in Matmata in Tunisia and Cappadocia in central Turkey [2].

In recent years several investigations were performed and showed that there can be multiple solutions to the excessive heat problem. Popular is cooling ventilation using a solar chimney [2, 6, 7]. The results showed that cooling ventilation using a solar chimney can reduce internal temperatures of buildings. Shading devices (overhangs and verandas) to reduce summer solar radiation are also investigated and useful depths of these shading elements for various orientations in continental climates were defined [8].

Space cooling can also be achieved by improving of the performance of roofs. This is because the roofs are the most exposed surfaces to direct solar radiation and can cause excessive heat gain in hot periods. Some efforts were made by investigators to improve the roof thermal performance. The use of low emissive material in the attic of a roof reduced the underside ceiling surface temperature which lowered the room air temperature [9]. Evaporative cooling approach for passive cooling of buildings in hot arid climates has also become an attractive subject of investigation for many researchers. The relative advantage of evaporative cooling in relation to many other approaches (cavity wall, insulation, white wash and large exposure orientations, vegetable pergola shading, roof with removable canvas, water film , soil humid grass and roof with white pots cover) were demonstrated by [10, 11].

The reduction of heat gain through the roofs using evaporative cooling systems was extensively investigated on open roof pond [18, 19], on water spraying over the roof, moving water layer over the roof, thin water film and roofs with wetted gunny bags [12-17]. Chandra and Chandra [12] have developed a periodic heat transfer model to study the effects of evaporative cooling using water spray and variable ventilation on the temperature control of a non-air-conditioned building. The influence of evaporative cooling over the roof as compared to bare roof case and intermittent ventilation as compared to the continuous or no-ventilation case have been assessed for controlling the indoor air temperature. It was found that the effectiveness of the evaporative cooling can be improved by conscious choice of the rate and duration controls the inside air temperature significantly. It was concluded that a combination of evaporative cooling and variable ventilation can make the internal environment of a building more comfortable. Chandra et al. [13] presented a theoretical assessment of three roof cooling systems for a non air conditioned building, and showed that the maximum cooling is achieved by water spray over the roof. But the roof pond system with stationary water is more effective in stabilizing the fluctuations of indoor temperature.

The present study suggests an improved roof design by combining the advantages of previous described cooling techniques (water ponds, low emissive surfaces) and inserted rocks of high thermal capacity materials. The resulting design can be more advantageous and effective than other systems for reducing heat during daytime and storing coolness at night. High thermal capacity materials (rocks bed) will delay the entry of daytime heat into the building by such a period that it reached the interior during the evening when it is least bothersome or often welcome. The roof is composed of a concrete ceiling and a flat aluminium plate separated with air space partially filled with rocks inserted in small quantity of water. The system is closed to prevent water vapour skipping outside. A schematic diagram of the model design is shown in figure 1.

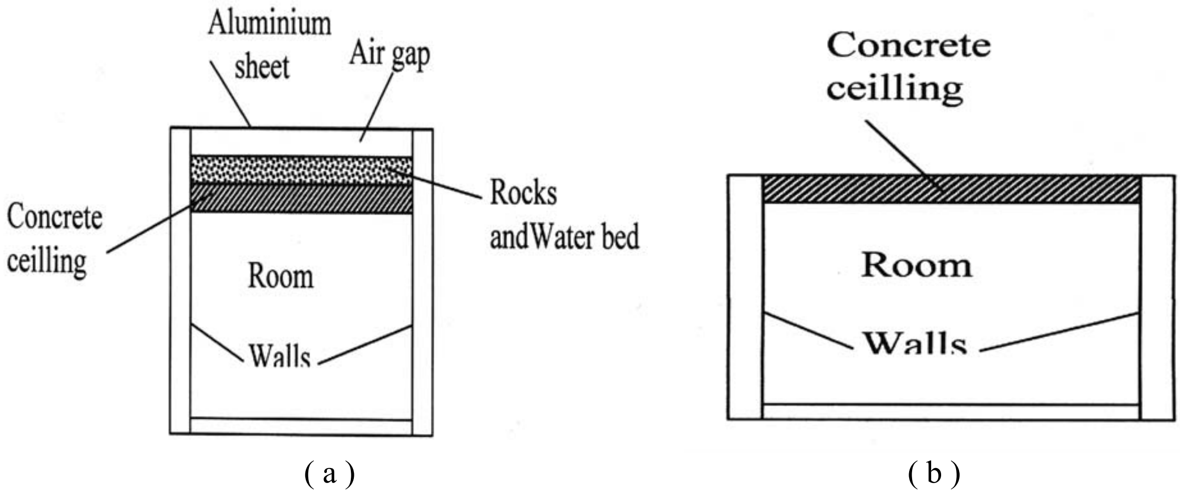

Fig. 1: Model description
(a) room with cooling roof system
(b) room without cooling roof system

2. MATHEMATICAL MODEL

The basic configuration of the model considered here are shown in figure 1. It is a cubic room with 3m high and 3m wide. South wall is provided with a window and the North one is provided with a door. Physical properties of materials used for the roof are presented in table 1.

Table 1: Materials properties

Element	Material	Thickness (m)	Density (kg/m^3)	Specifique heat (J/kg.K)	Conductivity (W/m.K)
Roof	Concrete slab	0.10	2400	1080	1.8
	Rocks	0.10	2600	800	2.3
	Water	0.07	1000	4175	0.613
	Aluminium	0.005	2750	936	204
Walls	Concrete slab	0.20	2400	1080	1.8

The purpose of the present mathematical model is to determine the inside air temperature at each time step as a function of outside air temperature, solar radiation and heat due to ventilation. The solution is based on the inside heat balance at each time step, the method of lagging with zone capacitance uses information from previous time steps to predict system response and update the zone temperature at the current time. 15 minutes is used as a time step (the shorter the time step the smaller the error). The simulation was done for the described model for two highest temperature summer days. The model was assumed situated in Algeria (Laghouat, latitude +33.46°, longitude +2.56° and elevation 767 m). The simulated day was the 26 of July, the maximum and the minimum temperature were respectively 42.7°, 24.5°.

2.1 Inside air heat balance equation

The heat balance for the inside room air may be formulated as follows :

$$C_{ai}\,\frac{d\,T_{ai}}{d\,t} = Q_{int} + Q_{ci} + Q_{v} \tag{1}$$

where $C_{ai}\,\frac{d\,T_{ai}}{d\,t}$ is the change in the energy stored (heat contents) of inside room air.

C_{ai} is the thermal capacitance of inside air (JK^{-1}).

Q_{int} is the total internal heat load from light and occupants. In the present model this load is neglected.

Q_{ci} is the total convective heat gain term from the inside room surfaces (walls, ground and roof) and may be expressed as :

$$Q_{ci} = \sum_{i=1}^{6} h_i\,A_i\left(T_{si} - T_{ai}\right) \tag{2}$$

Q_v is the heat transfer due to air ventilation term which can be expressed as :

$$Q_v = m_{ae}\,C_{ae}\left(T_{ae} - T_{ai}\right) \tag{3}$$

The derivative term of $C_{ai}\,\frac{d\,T_{ai}}{d\,t}$ can be expressed by a finite difference approximation as follows :

$$\frac{d\,T_{ai}}{d\,t} = \left(T_{ai}^{t} - T_{ai}^{t-\Delta t}\right)\frac{1}{\Delta(t)} \tag{4}$$

by replacing equations (1) to (4) into equation (1) we can obtain inside air temperature as follows :

$$T_{ai}^{t} = \frac{\sum Q_i^t + \left\{C_{ai}\,\frac{T_{ai}}{\partial t} + \sum A_i\,h_i\,T_{si} + m_{ae}\,C_{ae}\,T_{ae}\right\}^{t-\partial t}}{\frac{C_{ai}}{\partial T} + \sum\left(A_i\,h_i + m_{ae}\,C_{ae}\right)} \tag{5}$$

The unknown mean inside air temperature, T_{ai}^{t}, is expressed as a function of inside surface temperatures and external air temperature at each time step t.

2.2 Surface temperatures

To calculate the internal surface temperatures, T_{si}^{t}, at each time step, t, as function of outside conditions, finite difference equations based on heat balance at each node was used, which allows for temperature determination at any point of interest. The first step is to select these points, by subdividing the medium into number of small layers presented by reference points called nodes. In our case we considered the heat flow in one direction in plane elements (walls, roof and floor) composed with deferent materials, so each material is divided into small layers and presented by nodes, Clarke suggested that three nodes per homogeneous element and one hour time step, in buildings application are consistent with acceptable accuracy [20]. The temperature for each single node at time t is evaluated by heat balance equations. The inter heat exchange between internal slab nodes are modelled using Fourier's one dimensional heat conduction equation [21].

$$\frac{\partial T}{\partial t} = \lambda \frac{d^2 T}{\rho_c \, d x^2} \tag{6}$$

This equation can be solved numerically by dividing the element into layers of thickness Δx called node, making a heat balance for each node at mid element, the heat balance equation for plane element composed with non homogeneous materials.

The boundary condition for the inside surface nodes in contact with room air may be given by :

$$\frac{\lambda \partial T_{si}}{\partial x} = h_i \left(T_{si} - T_{ai}\right) \tag{7}$$

The boundary condition for the outside surface nodes in contact with outside air may be formulated using the following equation:

$$\frac{\lambda \partial T_{se}}{\partial x} = h_e \left(T_{se} - T_{ao}\right) \tag{8}$$

where h_i and h_e are the inside and outside combined convection and radiation coefficients whose values according to [22] are $h_i = E h_r + h_{ci}$ and $h_e = E h_r + h_{ce}$ and $h_{ce} = 0.76\ V + 2.8$, $h_r = 4\,\sigma\, T^3$, $E_{1,2} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2}}$.

The upper roof surface exchange heat with the outside air by convection and by radiation with the sky. According to [23], a horizontal surface with emissvity ε_r and absolute temperature T_r, produces a net radiative cooling rate Q_r, where ;

$$Q_r = A \sigma \varepsilon_r \left(T_r^4 - T_{sky}^4\right) \tag{9}$$

where $T_{sky} = \varepsilon_{sky}^{1/4}\ T_{ae}$, $\varepsilon_{sky} = 0.741 + 0.0062\,T_{dp}$ and σ is the Stephen Boltzman constant $5.67\ 10^{-8}$ W/m^2 K^4.

T_{dp} is the surface dew point temperature in °C, it was computed as a function of the ambient temperature (T_{ae}) and the relative humidity (RH), using the expression by Murray [24].

$$T_{dp} = 237.3 \frac{\ln RH + a.b}{\left(a - \mathrm{Ln}\,RH\right) + a.b} \tag{10}$$

where $0 \le RH \le 10$, $a = 17.2693882$ and $b = \frac{T_{ae}}{T_{ae} + 273}$

The heat exchange by convection for outside horizontal surface is given by :

$$Q_c = A\, h_{ce} \left(T_r - T_{ae}\right) \tag{11}$$

The heat exchange between the lower aluminium surface and the upper water rocks surface is by radiations, convection and evaporation. Following equations reported by [25] we may write :

- Heat exchange by radiations is given by :

$$Q_r = A\, h_{wr,al} \left(T_{wr} - T_{al} \right) \tag{12}$$

where $E_{wr,al}$ is the surface emissivity between water rocks and aluminium and is given by

$E_{wr,al} = \dfrac{1}{\dfrac{1}{\varepsilon_{al}} + \dfrac{1}{\varepsilon_{wr}}}$ and $h_r = 4\,\sigma\, T_{wr}^3$ and A is the area (m²)

- Heat exchange by convection is given by :

$$Q_c = A\, h_{c,wr,al} \left(T_{wr} - T_{al} \right) \tag{13}$$

where :

$$h_{c,wr,al} = 0.9 \left\{ \left(T_{wr} - T_{al} \right) + \left(\frac{P_{vs}\left(T_{wr}\right) - P_{vs}\left(T_{al}\right)}{273 - P_{vs}\left(T_{wr}\right)} \right) T_{wr} \right\}^{1/3} \tag{14}$$

- Heat exchange by evaporation and condensation is given by :

$$Q_{evp} = 6.3\,10^{-3} \left[P_{vs}\left(T_{wr}\right) - P_{vs}\left(T_{al}\right) \right] \times L \times h_{c,wr,al} \tag{15}$$

where L is the latent heat by evaporation at average temperature which is equal 2350 kJ/kg.
P_{vs} is the saturated vapour pressure in kPa at temperature T in °C.

For temperature range $20 \leq T \leq 80$ °C the below polynomial gives acceptable results [25]:

$$P_{vs}(T) = -16.037 + 1.8974\,T - 0.0699\,T^2 + 0.0012\,T^3 - 5.8511\,10^{-6}\,T^4 \tag{16}$$

In modelling the floor elements the earth temperature at 60 cm of depth below the floor is considered constant and equal to the daily average temperature of the region [2].

In the above equations the number of the unknowns is greater than the number of equations; these equations were solved by proposing the initial inside air temperature at start time t. This initial temperature T_{ai} (t) will not be correct and it is necessary to simulate the model with the same daily repetition of air temperature and solar radiation until the temperature of each node returns to the same value at the same time in each day simulation. At this point the building is in thermal harmony with the environment.

3. RESULTS AND DISCUSSION

Analysis of the results shows that the most significant factors affecting the cooling power of the passive cooling roof were the rocks, water volume, the aluminium roof thickness and the roof air space width. The simulation was done for two models, room with cooling roof system as shown, and room with bare roof. The model was assumed to be located in Laghouat city in Algeria at latitude +33.46° and Longitude +2.56° for an elevation 767m above see level. The simulated day was the 26 of July, the maximum and the minimum temperature were respectively 42.7° and 24.5°. Figure 2 shows comparison of room air temperatures with cooling roof system and with bare roof without room ventilation. It can be seen from this figure that the evaporative reflective roof can reduce the internal room air temperatures during the day up to 8 °C in comparison to the air temperatures for a bare roof over the room. Figure 3 is the comparison of room air temperatures with cooling roof system and with bare roof when room ventilation is allowed. The ventilation was allowed from 8 pm till 9 am, period when the outside air temperature is relatively cool. This can significantly improve cooling of room air temperatures as can be shown from figure 4.

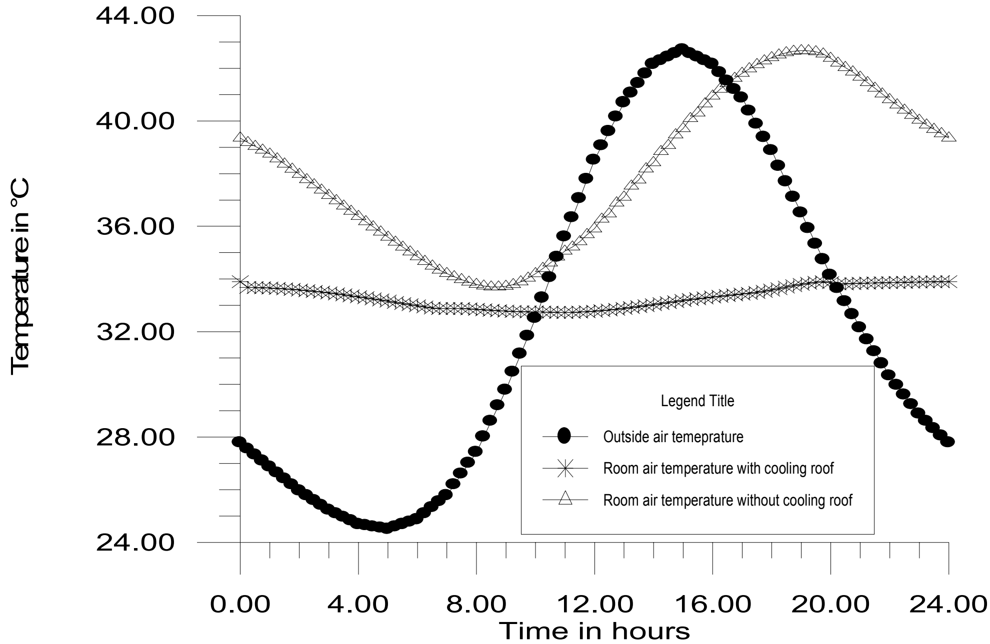

Fig. 2: Comparison of room air temperatures with cooling roof system and with bare roof (for non ventilated room case

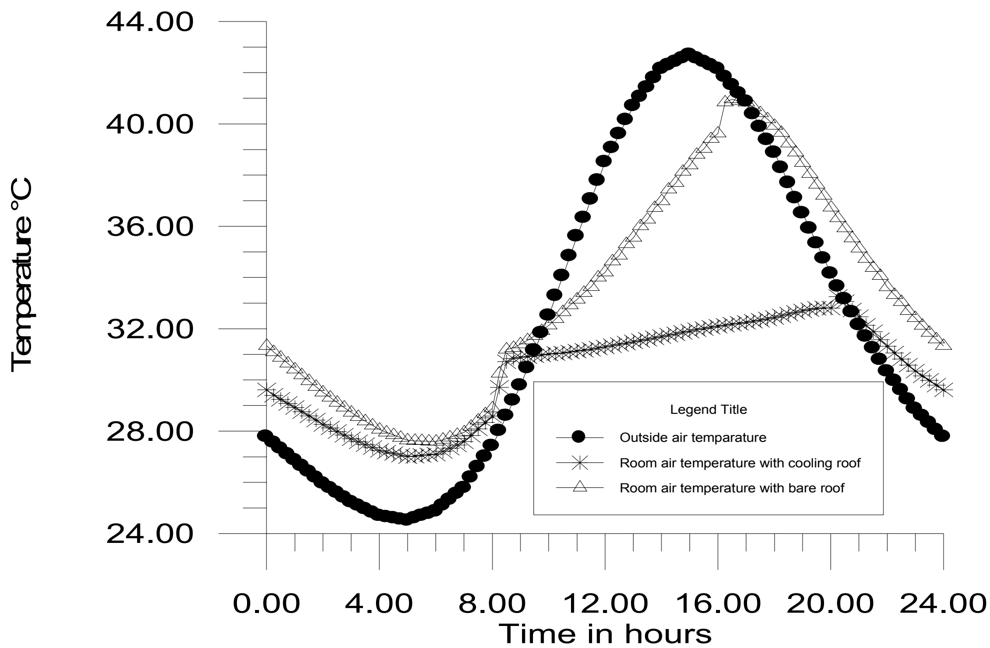

Fig. 3: Comparison of room air temperatures with cooling roof system and with bare roof (for ventilated room case)

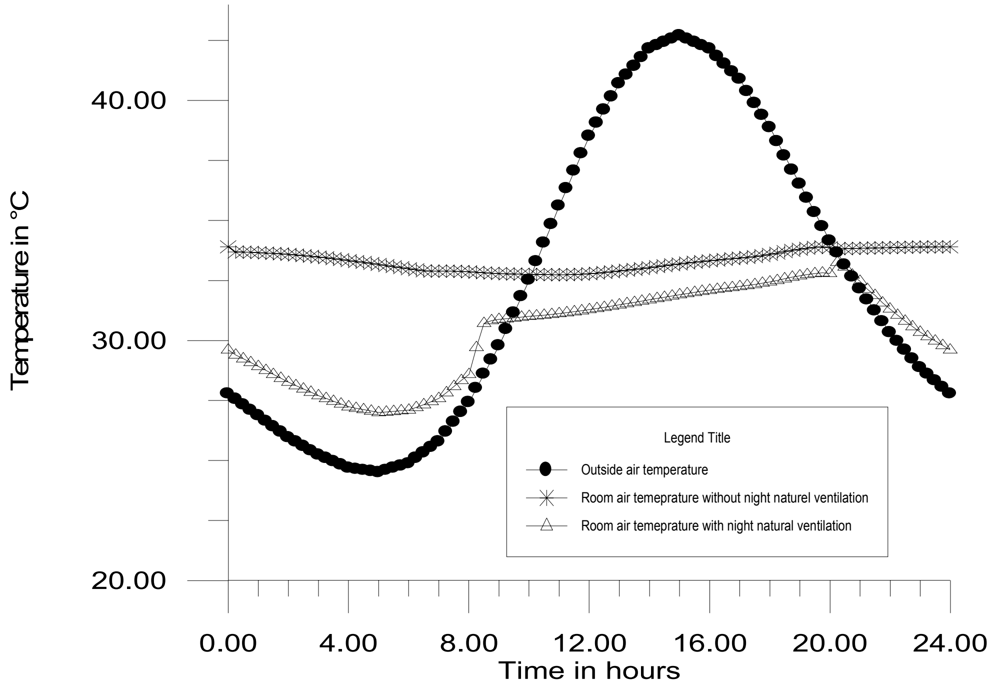

Fig. 4: Comparison of room air temperatures with cooling roof system (for ventilated and non ventilated room case)

4. CONCLUSION

A dynamic mathematical model for an evapo-reflective roof used to improve space cooling in buildings for hot arid climates has been developed. The analysis examined theoretically the effectiveness of such roof cooling system in comparison to a bare roof. The results showed that cooling inside buildings can be improved by the application of such cooling design. It was also seen that combining evapo-reflective roof with night ventilation increase more significantly such cooling.

High specific heat capacity (C_p) materials were used such as a bricks in the walls and rocks water in the roof increases the time lag and delay the maximum indoor temperature until the outside air temperature is acceptable, which is shown in figure 2.

NOMENCLATURE

C_{ai} : Thermal capacitance of inside air (JK^{-1})

C_{ae} : Specific heat of outside air ($Jkg^{-1}K^{-1}$)

E : Surface emissivity

I : total solar radiation (Wm^{-2})

I_j : Long wave radiation (Wm^{-2})

h_{ci} : Inside convection heat transfer coefficient ($Wm^{-2}K^{-1}$)

h_{ce} : Outside convection heat transfer coefficient ($Wm^{-2}K^{-1}$)

h_r : Radiation heat transfer coefficient ($Wm^{-2}K^{-1}$)

$h_{c,\,wr,\,al}$: Convection heat transfer coefficient between the rock bed and aluminium ($Wm^{-2}K^{-1}$)

P_{vs} : Saturated vapour pressure (kPa)
T_{al} : Aluminium outside surface temperature (°C)
Tao : Solar air temperature (°C)
T_{si} : Inside surface temperature (°C)
T_{se} : Outside surface temperature (°C)
T_{wr} : Rock bed upper surface temperature (°C)
V : Wind speed (ms^{-1})

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