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Fatigue Estimation for a Rotating Blade of a Wind Turbine

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Abstract – *The fatigue estimation of a rotating blade can be useful for preventing blade breakage, which is a very frequent problem in the use of wind turbine.*
The estimation of fatigue for a rotating blade must go through several steps of calculation: among them the calculation of mode shapes and frequencies and the computation of displacements and stresses acting on the blades.
In this work two methods are used in the calculation of the mode shapes, the first one is based on the discretization of the blade element, whereas the second method is based on the resolution of the differential equation of the blade as a continuous system.
This later method is more advantageous in determining mode shapes as well as stresses acting on the blades because it doesn't require the use of a powerful computer.
Once the stresses are calculated, the fatigue is estimated using the theory of « cumulative damage in fatigue ». This estimation must be based on the statistical distribution of wind speed for the site in use.

Résumé – *Le calcul de la fatigue d'une pale rotative peut nous aider à prévoir les problèmes de structure généralement rencontrés dans l'utilisation des aérogénérateurs (éoliennes rapides), où la rupture des pales est très fréquente. Ce calcul doit passer par plusieurs étapes parmi elles le calcul des fréquences et des modes propres ainsi que le calcul des déplacements et des contraintes qui agissent sur ces pales. Dans ce travail deux méthodes différentes sont utilisées pour le calcul des modes propres :*
la première méthode est basée sur la discrétisation de la pale, la seconde est basée sur la résolution de l'équation différentielle de la pale comme un système continu.
Cette dernière méthode est plus avantageuse dans le calcul des modes propres ainsi que dans le calcul des contraintes, car elle ne nécessite pas l'utilisation d'un calculateur puissant.
Finalement, la fatigue a été estimée en utilisant la théorie de Miner du dommage cumulé et en se basant sur la distribution statistique des vitesses extrêmes du vent pour le site en question.

Keywords: Wind Energy - Structural dynamics - Aerodynamics - Numerical Analysis.

1. INTRODUCTION

The use of the wind energy to generate electricity is a twentieth –century development. The first oil price shock, in the early 1970s, stimulated interest in alternative energy sources, the research and development in wind turbine technology received a substantial boost in several countries. Also the increasing concern for the environment provided a further impetus for cleaner sources of energy, including wind energy. These two factors promoted interest in the large-scale generation of electricity by wind power [1].

The major cause of wind turbine failure is fatigue, because wind turbine blades exposed to wind loading are vulnerable to cumulative fatigue damage owing to the cyclic nature of the loading. The difficulty in predicting fatigue is in large part due to an insufficient knowledge of the dynamic behavior of these machines. Moreover, S-N curve that gives the number of stress cycles to failure is encumbered with uncertainty owing to a limited number of test specimens as well as variability from one specimen to another [2].

The prediction of the dynamic behavior constitutes one of the most important processes in the design of wind turbines, because it can be useful in estimating the energetic performance of the wind turbine as well as the fatigue and the structural problem of this machine. The study of this dynamic behavior can be undertaken by various analysis methods [3].

The estimation of fatigue for a rotating blade must go through several steps of calculation. The first step is the calculation of mode shapes and frequencies, which is a difficult task due to the complicated nature of the blade rotating movement. The approach used in this work is based on the study of the blade movement. For simplification this movement is decomposed into two: bending and torsion. Each movement is then studied in a separate manner.

For bending natural frequencies are determined using a discrete method. This method is based on the equilibrium equations of the blade elements. After assembling of all the elements the equilibrium equations can be transformed into a matrix relation. The resolution of this matrix relation gives the frequencies and the mode shapes of the blades. The mode shape curves obtained by this method are not sufficiently precise, due to the insufficient number of points determined, and therefore cannot be used in the stress calculation. For this reason the mode shapes are then recalculated using another method, based on the numerical resolution of the mode shape differential equation, despite the difficulties encountered in solving its specific boundary conditions.

This later method is more advantageous in determining mode shapes than the discrete method (matrix method), because it allows us to determine the mode curves in a more precise and continuous manner (large number of points are determined) without having to use a powerful computer. For each method a Fortran computer program is elaborated.

For torsion the equation of the movement is solved in a similar way as in the case of bending to determine the torsional mode shapes and frequencies.

Finally the two movements are coupled using the equation proposed by Brooks. This equation is resolved iteratively in order to determine the displacements and stresses acting on the blade.

Another computer program is written for the coupled equation resolution. If the matrix (discrete) method is largely used, the originality of this work is characterised in the numerical resolution of the blade equations, using it as a continuous system. This is done in the case of the mode shape calculation, and also in solving the coupled equation (bending-torsion), in order to determine displacements and stresses acting on the blade.

Once the stresses are calculated, the fatigue is estimated using the miner theory of « cumulative damage in fatigue ». This estimation must be based on the statistical distribution of wind speed for the site in use.

In this work only a horizontal axis wind turbine is considered, since their use now is almost universal.

2. CALCULATION OF BENDING FREQUENCIES:

The equilibrium equations for a blade element having a length l, are given [4]:

$$G_{n+1} = G_n + \int_{x_{n+1}}^{x_n} \Omega^2.x.m.dx = G_n + 0.5.\Omega^2.m.(x^2_n - x^2_{n+1}) \qquad (1)$$

$$V_{n+1} = V_n - \int_{x_{n+1}}^{x_n} \omega^2.Z.m.dx = V_n - \frac{1}{2}.m.\omega^2.\,l.(Z_n+Z_{n+1}) \quad (2)$$

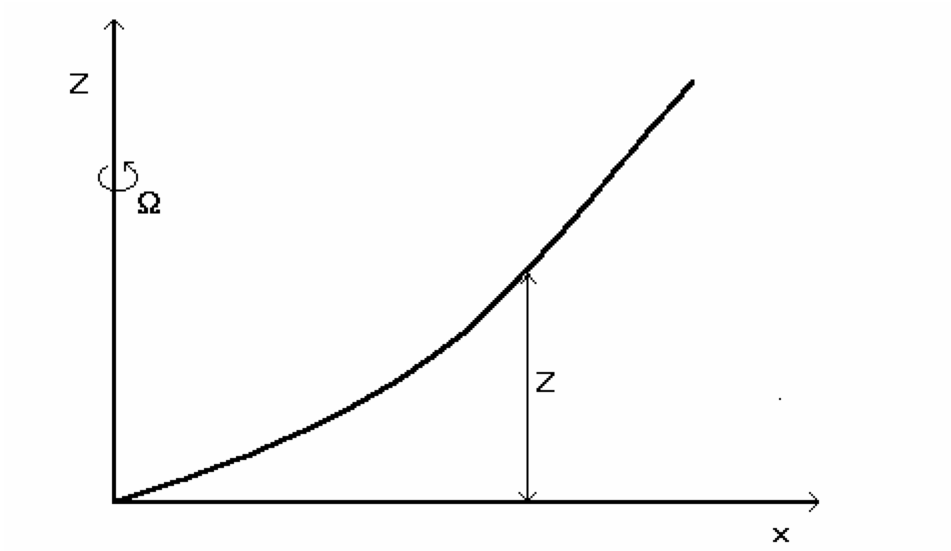


Fig. 1: rotating blade movement

$$M_{n+1} = M_n - G_n(Z_n - Z_{n+1}) - V_n.1 + \int_{X_{n+1}}^{X_n}(x - x_{n+1})\omega^2.Z.mdX - \int_{X_{n+1}}^{X_n}(Z - Z_{n+1})\Omega^2 X.m.dx \quad (3)$$

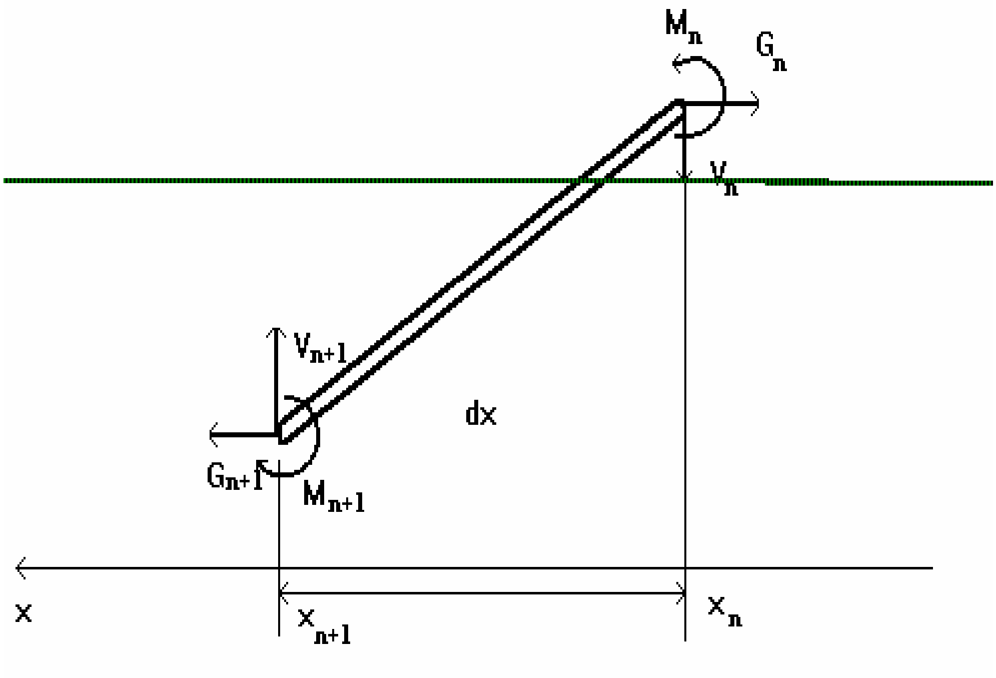


Fig. 2: Blade element equilibrium

Where:

V_n is the shear force at the node n
M_n is the bending moment node n
G_n is the centrifugal force at the node n
Z_n is the deflection of the node n
m is the linear mass
ω is the natural frequencies

After assembling all the blade elements the sequences V, G, M can be transformed to a matrix equation of the form:

$$(A - I / \omega^2) .Z = 0 \tag{4}$$

Where the eigenvector $\mathbf{Z_i}$ are the mode shapes , and the eigen values are proportional to the frequencies .
Figure 1 gives the first mode shape calculated using this method.

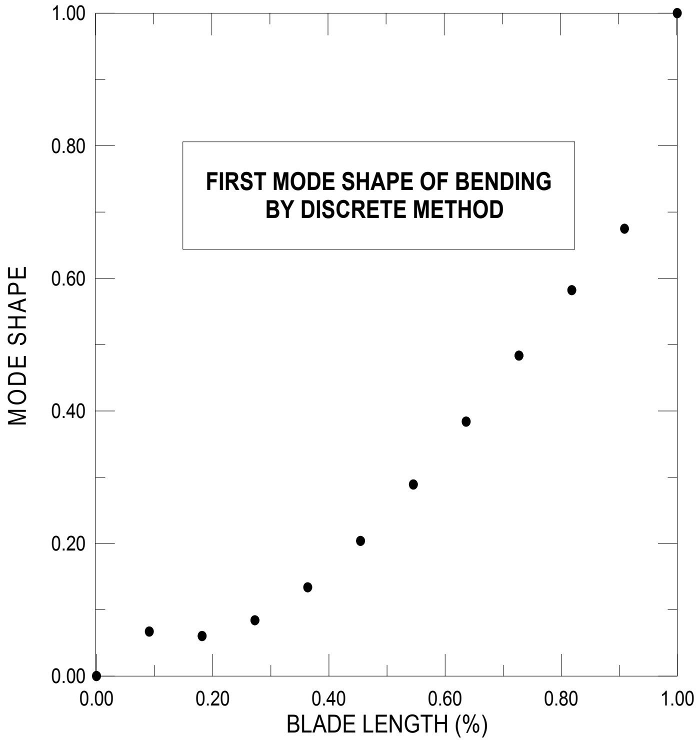


Fig. 3: The first mode shape using the matrix method

The use of this method in the mode calculation is limited by the capacity of the computer used. For this reason another method is used, and which is based on the numerical resolution of the mode shape differential equation, after substituting in it the frequencies obtained above.

3. RESOLUTION OF THE MODE SHAPE EQUATION

The bending movement is expressed by the following equation [4]:

$$\frac{\partial^2}{\partial x^2}(E.]\frac{\partial^2 Z}{\partial x^2}) - \frac{\partial}{\partial x}(G\frac{\partial Z}{\partial x}) + \frac{\partial^2 Z}{\partial t^2} = \frac{\partial F}{\partial x} \tag{5}$$

Where:

$$G = \int_x^L m.\Omega^2.x.dx$$

t is time .
F is the aerodynamic load .

In case of free vibrations, equation (5) can be written:

$$\frac{\partial^2}{\partial.x^2}(E.I\frac{\partial^2.Z}{\partial.x^2}) - \frac{\partial}{\partial.x}(G\frac{\partial.Z}{\partial.x}) + m\frac{\partial^2.Z}{\partial.t^2} = 0$$

(6)

Taking: $Z = S(x).\varphi(t)$ And using the method of variable separation, the set of two ordinary differential equations is obtained:

$$\frac{d^2}{dx^2}(E.I\frac{d^2.S}{dx^2}) - \frac{d}{dx}(G\frac{dS}{dx}) - m\omega^2 S = 0 \qquad (7)$$

$$\frac{d^2\varphi}{dt^2} + \omega^2\varphi = 0 \qquad (8)$$

Values of ω are taken from the precedent method. The boundary conditions of equation(7) are:

I. Fixed end:

$S(0) = 0$ (Displacement =0)

$\frac{dS(0)}{dx} = 0$ (Slope = 0)

II. Free end:

$\frac{d^2S(L)}{dx^2} = 0$ (Bending moment =0)

$\frac{d S^3(L)}{dx^3} = 0$ (Shear force = 0)

The difficulty of numerical resolution of equation (7) is due to these boundary conditions. To make this resolution possible, an algorithm is developed [5] in order to estimate the initial conditions equivalent to (II).

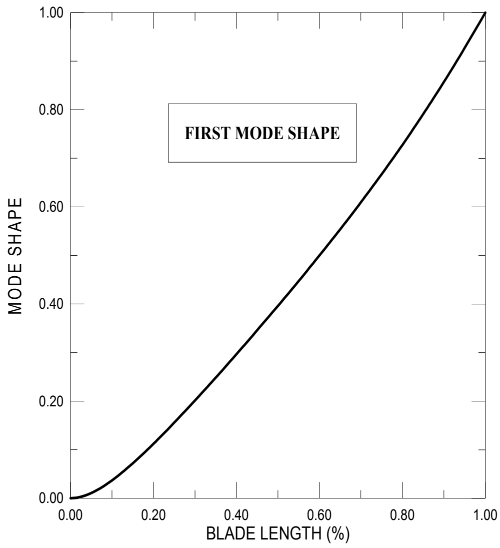


Fig. 4: The first mode shape using the second method

This algorithm is inspired from the shooting method, in which initial value of the function is supposed in order to calculate the final value, this later value allow the correction of the initial value till the right final value is obtained. But in our case the problem is more complicated since it has two initial values, at the fixed end and two final values, at the free end. The efforts deployed in this resolution represent the principal original contribution of this work.

The predictor corrector method is then is used to solve this equation, after the failure of Runge-Kutta method to reach convergence.

The mode shapes obtained using this approach are represented by Figures 4, 5 and 6.

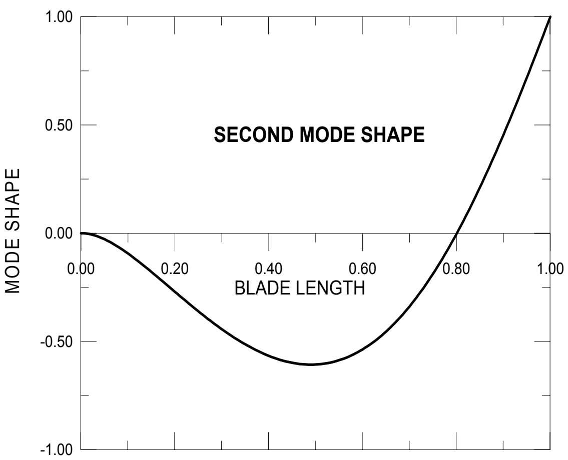


Fig. 5: The second mode shape using the second method

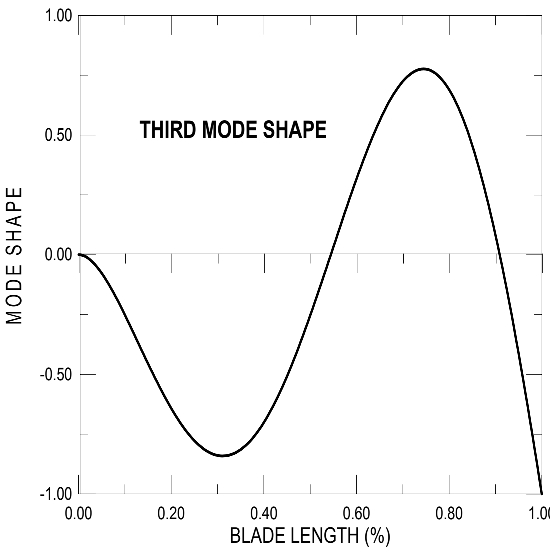


Fig.6: The third mode shape using the second method

This method is more advantageous then the matrix method, because it enables us to determine the mode shape curves in a smooth way. This is due to the fact that the number of the curve points calculated is not limited by the memory size, Since each point is calculated in function of the four precedent points (The predictor corrector method Adam's formula [6]). We then only need to keep in memory the values of the four last points.

4. TORSIONAL MOVEMENT

Torsional movement can be expressed by the following equation [1]:

$$\frac{\partial}{\partial x}(G.I\frac{\partial\theta}{\partial x}) - C.\frac{\partial^2}{\partial t^2} - C.\Omega^2.\theta = -\frac{\partial L_A}{\partial x} \qquad (9)$$

Where:

- C is the moment of inertia per unit length .
- Ω is the angular velocity .
- θ is the twist angle .
- L_A is the aerodynamic moment .
- G.J is the torsional rigidity .

Equation (9) can be solved in a similar manner as (5) (bending equation.)

5. EQUATION OF COUPLED MOVEMENT

The two equation (bending - torsion) must be coupled in order to determine stresses and displacements. This is done using the equation of Brooks [4] , which has the form :

$$\frac{\partial^2}{\partial.x^2}(E.I\frac{\partial^2.Z}{\partial.x^2}) - \frac{\partial}{\partial.x}(G\frac{\partial.Z}{\partial.x}) + m\frac{\partial^2.Z}{\partial.t^2} + Ft\,(x\,,\,\theta\,,\,\frac{\partial^2.\theta}{\partial.t^2}) = \frac{\partial.F}{\partial.x}$$

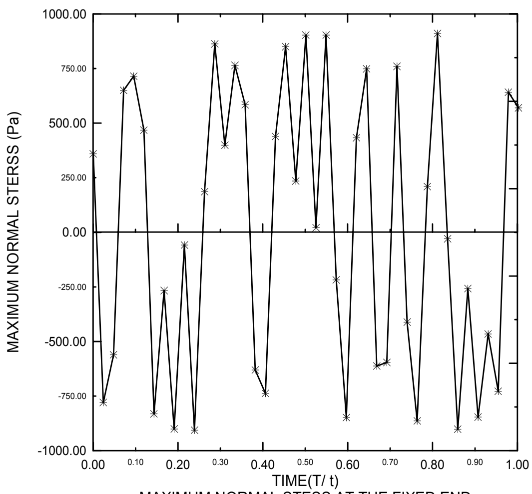
(10)

Where Ft is a known function.

In this equation the effect of torsion on bending is taken in account. The particular difficulty encountered in the resolution of the coupled equation is due to the fact that the wind load depends upon the shape of the blade (deflection), since this load is a function of the incidence angle, on the other hand the load deforms the shape of this blade. This interdependence between the aerodynamic load and the blade deflection is a source of nonlinearity that complicates the numerical resolution.

To overcome this difficulty an initial deflection is supposed, then an iterative method is used to correct this shape. In this resolution, the computation time can cause serious problem if adequate techniques are not taken.

Some results obtained are shown in Figure 7 and in Figure 8.



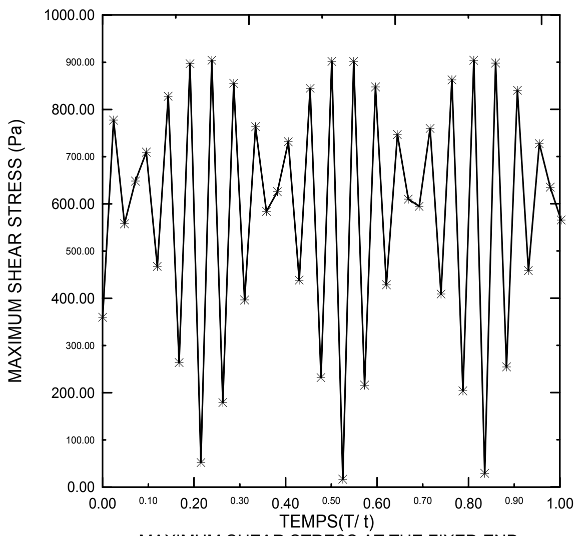


Fig. 7: Maximum normal stress at the fixed end.

Fig. 8: Maximum shear stress at the fixed end.

Figure 7 represents the maximum normal stress and Figure 8 represents the maximum shear stress for a wind speed of 3m/s.

6. FATIGUE CALCULATION FOR THE BLADE

The fatigue estimation for the blade is based on theory of Miner, which can be applied in the case of a machine part operating under alternative stress having a variable amplitude [7].

This theory is known as «the linear cumulative damage rule » or « Miner's rule. » This theory assumes that every operating cycle consumes a percentage of the part life. Hence the total life of the part can be estimated by adding up the percentage of life consumed by each overstress cycle. Miner theory is stated mathematically as follows: If stresses with amplitudes σ_1, σ_2, ... σ_k are applied to a part for a total number of cycles n_1, n_2, ..., n_k respectively and suppose that the lives (the allowable number of cycles) corresponding to these stresses are : $N_1, N_2, ..., N_k$, then failure may occur if :

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \cdots + \frac{n_k}{N_k} = 1 \qquad (11)$$

In a more recent article [8], Miner cites numerous tests that showed if the loading is random, equation (11) will usually give conservative predictions (i.e. $\sum_i \frac{n_i}{N_i} > 1.$)

In this work a two-blade wind turbine is used with a naca profile. The values of n_i are calculated for a life period equals to ten years and based on the statistical distribution of wind speed (Figure 9), while the values of N_i are taken from the endurance limit curve.

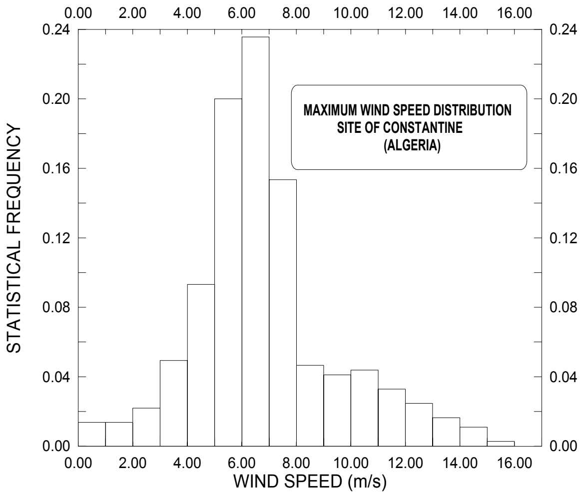


Fig. 9: extreme wind speed statistical distribution

7. RESULTS

The Characteristics of the turbine used are:

- Number of blades: 02
- Profile: NACA0012

- Material used: Aluminum alloy
- Blade length: 8m
- Average cord: 0.4 m
- Specific velocity (velocity ratio): 8.

The results obtained using these blade Characteristics are:

- The first mode shape calculated by the discrete method is given by Figure 3.
- The first three mode shapes calculated by the numerical resolution of the differential equation are given by Figures 4 to 6. These curves were compared with those obtained by reference [9] and the two results were found to be very similar.
- It is noticeable that Figures 4, 5 and 6 give smoother and more precise mode shapes than Figure 3.
- Figures 8 and 7 represent the maximum normal stress and the maximum shear stress in function of time, for a wind speed of 3m/s. These stresses are determined by the numerical resolution of the coupled equation (bending – torsion.)
- For other range of wind speed (6 , 9 , 12 and 15m/s) similar curves are determined.
- To estimate the fatigue, the value $\sum_i \frac{n_i}{N_i}$ is found to be equal to 0.8 (less then unity).
- The values of n_i are calculated for a life time equals to ten years and determined from the statistical distribution of wind speed (see Fig. 9), while the values of N_i are taken from the endurance limit curve.

8. CONCLUSION

The fatigue estimation can be helpful in preventing blade breakage, which is a very frequent problem in the use of wind turbines.
The stresses calculated by the coupled equation are used to estimate the fatigue of the blade by the cumulative damage theory. In this theory the wind speed statistical distribution for the region of Constantine is used, in order to determinate the number of operating cycles for each wind speed (stress amplitude.)

According to this theory the blade can resist for an operating period close to ten years, with a maximum operating wind speed of 12 m/s. Although the characteristics of the blade chosen are not optimal, the above calculation may be used to develop a mathematical model to optimize these characteristics, in order to minimize its structure problems and maximize its energetic performance.

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