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Research Paper

Study of two-dimensional transient heat transfer through a multi-layers wall: Application to the buildings thermal insulation

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A B S T R A C T

The two-dimensional transient heat conduction through a multilayers wall made of different materials and thicknesses was numerically resolved. The equations system resolution was carried out by Alternating direction implicit method (ADI). The outdoor and indoor temperatures and the convective coefficients used as boundary conditions in the developed Fortran program were from the Algerian regulatory technical document. After validation of the Fortran program with literature, it was used to studying the influence of different boundary conditions (bottom and top sides), on the thermal insulation in the building, for many configurations of external walls, usually used in building construction at Batna city (Algeria). Results showed that for the configurations that give bad thermal insulation, the conditions imposed on the top and bottom of the wall have practically no influence on the internal temperature of the multilayers wall, however, the opposite is observed for the configurations that ensure good thermal insulation.

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1. Introduction

The exploitation of natural resources without affecting the natural balance is one of the major challenges of humanity. The production and consumption of energy contribute to global warming, reducing energy consumption is the most effective way to save energy and reduce pollution.

Buildings account for about 40% of the global energy consumption and contribute over 30% of the CO2 emissions, a large proportion of this energy is used for thermal comfort in buildings [1]. The global contribution from buildings towards energy consumption, both residential and commercial, has steadily increased reaching figures between 20% and 40% in developed countries and has exceeded the other major sectors, energy use is particularly significant in heating, ventilation and air-conditioning systems, it was 50% of building consumption and 20% of total consumption in the USA [2]. In Algeria between 2013 and 2015 it was 16.2% for electricity and 15.96% for gas. The residential sector consumes around 42.7% of total energy consumption, which gives an increase in consumption in this sector of 13.49%[3]. In 2008 India's energy use was the fifth highest in the world, the building sector represents about 33% of electricity consumption[4]. In Saudi Arabia, energy demand for residential buildings is of a very high level, whereby approximately 70% of electricity was consumed by air conditioning systems alone for interior cooling throughout the year due to the hot and humid climate [5]. Upward trend in energy demand will continue in the future due to the growth in population, increasing demand for building services and comfort levels, as well as a rise in time spent inside buildings.

Thermal insulation is one of the most effective ways to save energy in this sector, unfortunately, the priority of many countries is to meet the growing demand for housing, and reduce their costs without considering insulations [6].

Several studies either theoretical and experimental have been carried out to develop and improve thermal insulation in the building sector[7-11].

Daouas et Al.[12] presented a study of one-dimensional transient heat conduction through a composite wall consisting of N parallel layers of different materials and thicknesses, they used an analytical method, based on Complex Finite Fourier Transform to estimate the yearly cooling transmission loads for two types of insulation materials and two typical wall structures. Estimated loads are used as inputs to a life-cycle cost analysis in order to determine the optimum thickness of the insulation layer. A numerical finite volume model was developed and validated against the experimental results by Gounni et Al [13]. The numerical model is used to study the thermal performance of an external building wall outfitted with a newly developed insulation materials and submitted to the real climatic conditions of Casablanca, Morocco.

Other researchers used a commercial software, a comparative assessment of internal versus external thermal insulation systems for energy efficient retrofitting of residential buildings is performed by using the TRNSYS software [14]. Knarud and Geving [15] conducted a 2D numerical hygrothermal study on an internally insulated masonry wall with a smart vapor barrier using COMSOL Multiphysics and WUFI. The focus has been on comparing and discussing the options available in, and the challenges faced with. Ran and Tang [16] established a model using Design-Builder software. The model with green roof was simulated in three cases: no ventilation, night-time ventilation and night-time ventilation combined with walls insulation. They found that with the improvement of external walls insulation, the cooling effect of the night ventilation becomes more obvious for the extensive green roofs.

The two dimensional heat transfer equation has been numerically resolved using different methods, Mansur et al [17] presented a new algorithm for the numerical solution of the linear 2D heat conduction equation using the 'Green Explicit' Approach (AGEx), the method employs the Green matrix which represents the domain of the problem to be solved in terms of physical properties and geometric characteristics. The temperature field is explicitly calculated by the Green matrix (GM) then it is determined by the Finite Element Method (FEM). The AGEx numerical results are quite precise compared to the analytical solutions and to the numerical solutions obtained by the FEM. Filippo et al [18] published a work in which they determined and simulated analytical solutions modelling transient heat conduction processes in 2D Cartesian finite bodies for small values of time. A 2D semi-infinite problem is solved explicitly and evaluated numerically as part of the analysis. The possibility of using approximate solutions to the heated surface of a certain basic one-dimensional geometry in a short time step was examined and discussed. Dutta [19]used the finite volume method to study heat transfer through a 2D multi-solid wall for different physical characteristics and for different boundary conditions (constant temperature, isolated wall, convective exchange, Constant flux] as well as for different values of initial temperature.

Chaabane et al[20] presented the Boltzmann lattice method (BLM) which they applied in the resolution of heat transfer in two-dimensional geometry with mixed conditions, emphasizing the flexibility of the method (BLM) and its efficiency. To deal with any combination of boundary conditions. The results obtained by their numerical approach are compared with those available in the literature.

In this work, the two-dimensional transient heat transfers through a multilayer wall were numerically resolved using Alternating direction implicit method (ADI). The climatic conditions of an Algerian city were used as boundary conditions. After validation of the developed program, it was exploited to investigate the effect of the different studied parameter on the performance of thermal insulation for several configurations (presented in Table 1) usually used in the building of this city.

Table 1. Different studied configurations.

			Three layers		
	Plaster	Brick	Mortar		
Configuration 1	$e_1 = 2\ cm$	$e_2 = 15\ cm$	$e_3 = 2\ cm$		
Configuration 2	$e_1 = 2\ cm$	$e_2 = 20\ cm$	$e_3 = 2\ cm$		
Configuration 3	$e_1 = 2\ cm$	$e_2 = 30\ cm$	$e_3 = 2\ cm$		
			Five layers		
	Plaster	Brick		Brick	Mortar
Configuration 1	$e_1 = 2\ cm$	$e_2 = 10\ cm$	$e_3 = 5\ cm$ Air	$e_4 = 15\ cm$	$e_5 = 2\ cm$
Configuration 2	$e_1 = 2\ cm$	$e_2 = 10\ cm$	$e_3 = 5\ cm$ Polystyrene	$e_4 = 15\ cm$	$e_5 = 2\ cm$

2. Problem modelling

The present analysis is concerned with the two-dimensional transient heat transfer through a multilayers wall consisting of three or five layers of different materials and thickness (Fig. 1)

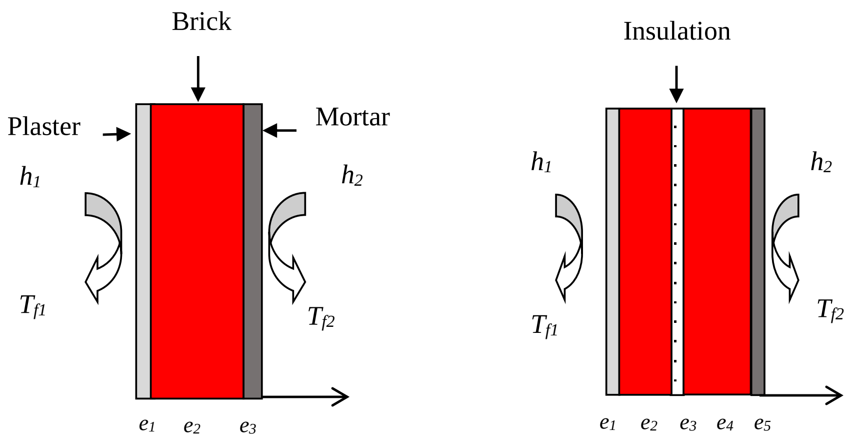

Fig 1. Schematic multilayer walls

The wall height is equal to 1m for all configurations. The different inputs data used in the developed program are presented in Table 1 and 2.

Table 2. Physical properties of different materials.

	Conductivity λ (W/m.K)	Density ρ (kg/m³)	Thermal capacity C_P (J/Kg. K)
Brick	0.6	1400	936
Mortar	1.4	2200	1080
Plaster	0.35	875	936
Polystyrene	0.04	16	1404
Air space	0.167	1.2	928

The internal and external side of the wall is submitted to a convective exchange with the ambient. The indoor and outdoor boundary conditions are at a fixed design value in winter, obtained from the Algerian regulatory technical document[21] (Table 3).

Table 3. Indoor and outdoor boundary conditions.

Temperatures	Convective coefficients
T_{f1} = 21 °C	h_1 =9 W/m² °C
T_{f2} = -4 °C	h_2 =16.5 W/m² °C

For the case the following modeling assumptions are made:

- Heat transfer in the wall is two-dimensional and unsteady.
- Physical properties are constants.
- There is no heat source in the wall.
- The contact between the wall layers is perfect.

The heat transfer through each layer of the composite wall is governed by the following heat conduction equation:

$$\alpha_i \frac{\partial^2 T_i}{\partial x^2} + \alpha_i \frac{\partial^2 T_i}{\partial y^2} = \frac{\partial T_i}{\partial t} \text{, with } \alpha_i = \frac{\lambda_i}{\rho_i \, C_{p_i}} \quad \text{and} \quad i = 1, \dots, 5 \tag{1}$$

$e_{i-1} \leq x \leq e_i$, $0 \leq y \leq H$, $t \geq 0$, $e_0 = 0$

With the following boundary conditions:

At left: for $x = 0$

$$-\lambda_1 \frac{\partial T}{\partial x} = h_1 \left(T_{f_1} - T\right) \tag{2}$$

λ_1 Thermal conductivity of the first layer.

At right, $x = L$

$$-\lambda_n \frac{\partial T}{\partial x} = h_2 \left(T - T_{f_2}\right); \quad n = 3 \; or \; 5 \tag{3}$$

λ_n Thermal conductivity of the last layer.

At the bottom side, $y = 0$

$$-\lambda_i \frac{\partial T}{\partial y} = H_1 \left(T_j - T_{j+1}\right) \quad , i = 1, \ldots, 5; \; j = 1 \tag{4}$$

At the top side, $y = H$

$$-\lambda_i \frac{\partial T}{\partial y} = H_2 \left(T_{j+1} - T_j\right) \quad , i = 1, \ldots, 5; j = m \tag{5}$$

Thermal contact between the layers is considered perfect. This condition is as follows:

$$\lambda_i \frac{\partial T_i}{\partial x} = \lambda_{i+1} \frac{\partial T_{i+1}}{\partial x} \quad \text{For } x = e_i \quad i = 1, .. n-1 \quad n = 3 \; or \; 5 \tag{6}$$

H_1 and H_2 are numerical coefficients when they are equal to zero the top side of the wall or the bottom one will be adiabatic, and when they have a high value we obtain the equality of the temperatures.

3. Numerical resolution

To solve Eq.1 with boundary conditions defined by Eqs. 2-6, the Alternative Implicit Direction Method (A.D.I.) was adopted. The principle of this method in Two-dimensional problems consists to solve the first dimension implicitly and the second dimension explicitly, and in the next step the first dimension explicitly and the second dimension implicitly, so forth. This allows saving in the storage and in the speed of resolution.

To discretize the Eq 1, the same step value along x and y axis ($\Delta x = \Delta y$) was considered, and with $r = \lambda \, \Delta t / \rho \; cp \; (\Delta x)^2$, the explicit scheme yields:

$$T_{i,j}^{k+1} - T_{i,j}^{k} = r \, (T_{i-1,j}^{k} - 2T_{i,j}^{k} + T_{i+1,j}^{k} + T_{i,j-1}^{k} - 2T_{i,j}^{k} + T_{i,j+1}^{k})$$

$$T_{i,j}^{k+1} = r \, (T_{i-1,j}^{k} + T_{i+1,j}^{k} + T_{i,j-1}^{k} + T_{i,j+1}^{k}) + (1 - 4r) T_{i,j}^{k} \tag{6}$$

The value of r must be little then $1/4$:[22]

$$\underbrace{T_{i,j}^{k+1} - T_{i,j}^{k} = r \left[\; T_{i-1,j}^{k} \right.}_{} - \underbrace{2T_{i,j}^{k} + T_{i+1,j}^{k} + T_{i,j-1}^{k+1}}_{} - 2T_{i,j}^{k+1} + \left. T_{i,j+1}^{k+1} \right] \tag{7}$$

$\partial^2 T / \partial x^2$ at beginning $\quad \partial^2 T / \partial y^2$ at end

The reversing of the order of derivations in the second time step gives:

$$T_{i,j}^{k+2} - T_{i,j}^{k+1} = r \left[\underbrace{T_{i-1,j}^{k+2} - 2T_{i,j}^{k+2} + T_{i+1,j}^{k+2}}_{\partial^2 T / \partial x^2 \text{ at end}} + \underbrace{T_{i,j-1}^{k+1} - 2T_{i,j}^{k+1} + T_{i,j+1}^{k+1}}_{\partial^2 T / \partial y^2 \text{ at beginning}} \right] \tag{8}$$

An example of the numerical discretization scheme is given in Fig. 2

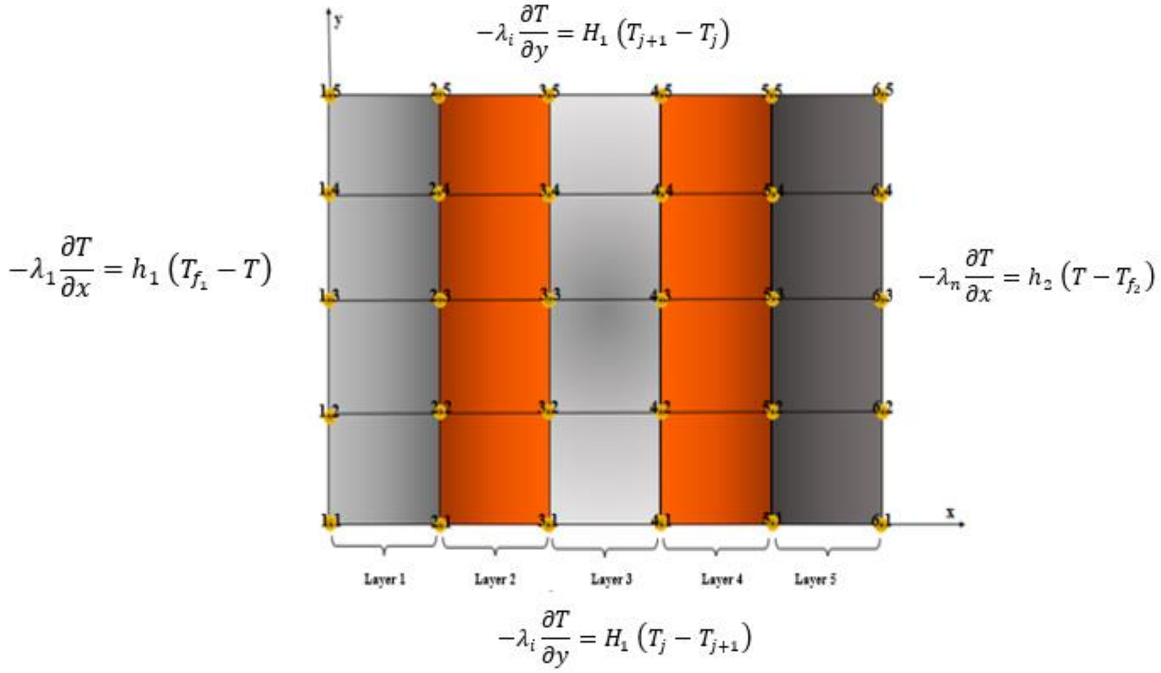

Fig 2. Discretization of five layers wall.

A Fortran program was developed according to the flow chart presented in Fig.3

The goal is investigating the influence of different parameters on the thermal insulation of the several studied configurations.

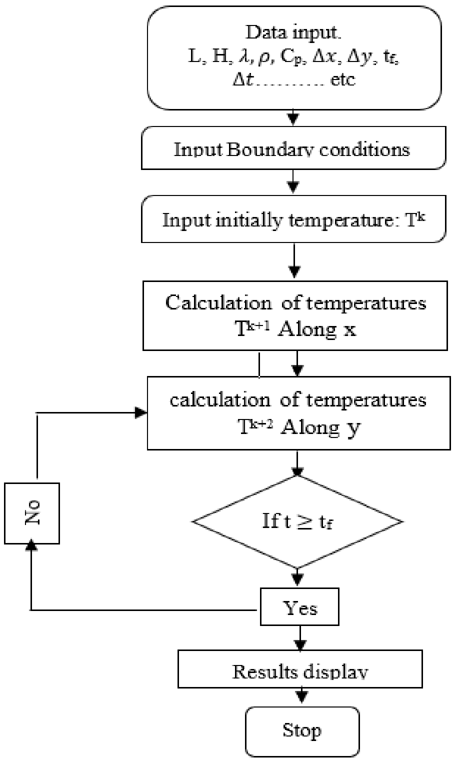

Fig 3. Flow chart of the developed program.

4. Results and discussion

Before presenting results, validation of the developed Fortran program would be appropriate, this was made first, by comparing results obtained using this program and those obtained by Mansur et Al [13], the case studied by the authors was a slab with 10 m long and 1 m wide, the physical parameters were $\alpha = 1m^2$ s and $\lambda = 1$ w / m ° C.

With an initial temperature of 0 ° C, the left side temperature was 0 ° C and the right side one was 1 ° C. The top and bottom sides are adiabatic, then the present program was run with the same conditions, results are shown in the Fig. 4, a good agreement can be observed.

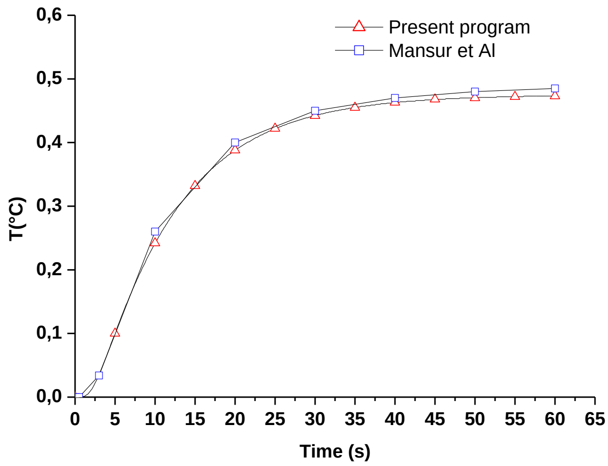

Fig 4. Temperature at a middle point of the slab left side versus time.

To ensure that the problem at the interface is rightly treated, we compared the results obtained at a point in the middle of the internal face for a wall made of three layers (10 cm thick each), composed of the same material with those obtained for the one layer's wall of 30 cm thick, a perfect agreement was obtained (Fig. 5).

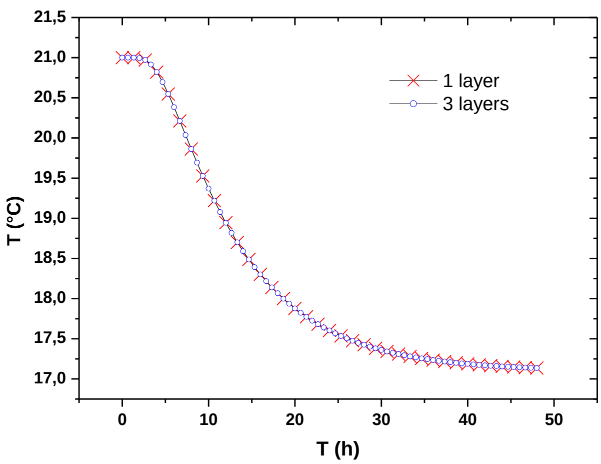

Fig 5. Temperature versus time at the middle of internal side.

Once the program validation carried out, several cases of the boundary conditions were studied (Table 4).

Table 4. Top and bottom boundary conditions

Cases	Adiabatic case	Case 1	Case 2	Case 3	Case 4
Conditions	For $y = 0$ and $y = H$ $\frac{\partial T}{\partial y} = 0$	$T_B = 21\ °C$ $T_T = -2\ °C$	$T_B = -2\ °C$ $T_T = 21\ °C$	$T_B = 21\ °C$ $T_T = 21\ °C$	$T_B = -2\ °C$ $T_T = -2\ °C$

For the five boundary conditions, the temperatures in the middle of the internal face for each configuration were shown in Fig. 6. It was observed that there is practically no influence of the top and bottom boundary conditions on the three layers wall (15 cm or a 20 cm of brick), the temperatures after 24 hours were respectively from 18.1 ° C to 18.2 ° C, and between 18.5 ° C and 18.7 ° C degrees. In the case of the three-layer wall made of 30 cm brick, and the case of five-layer walls with air gap or polystyrene, it was noted that the internal temperatures were sensitive to the top and the bottom boundary conditions. The temperatures after 24 hours were between 19°.25C and 19.75°C, this means that if the wall is well insulated the conditions on the top and bottom must be taken into account.

The two-dimensional field of temperatures was presented in Fig.7-11 for the three configurations that give good thermal insulation.

When the bottom and top sides are adiabatic (Fig 7), the three configurations give practically the same improvement in terms of thermal insulation.

In case 1, namely that when the bottom is heated and the top unheated, we notice that the temperature of the lower half of the wall was approximately 21 ° C, while the upper half approaches -2 ° C (Fig 8). Inversely, in case 2 It was noted that the temperature of the upper half of the wall is close to 21 ° C, while the lower half approaches -2 ° C (Fig 9)

When the top and bottom are heated (case 3), it was observed that the all internal face of the wall is at 21 ° C, (Fig 10) this means that the thermal insulation is more efficient in this case

On the other hand, if the top and the bottom are not heated (case 4). Fig 11 revealed that in the middle of the wall the temperature is between 17 ° C and 18 ° C in the case of a wall of 5 layers, whereas it is around 17 ° C for 3 layers, so the insulation is less efficient.

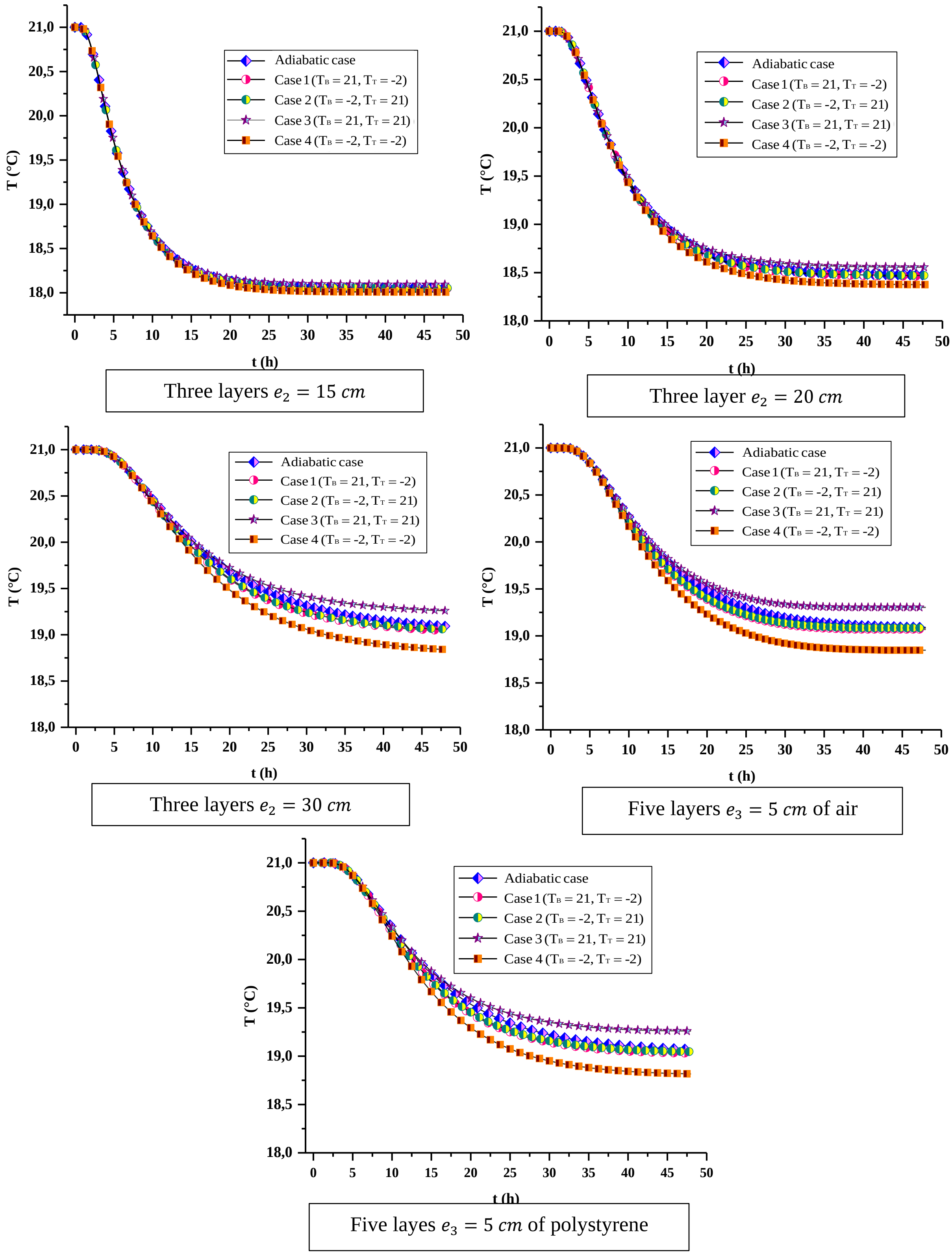

Fig 6. Temperature variation as a function of time in the internal plaster face for different configurations and for several cases.

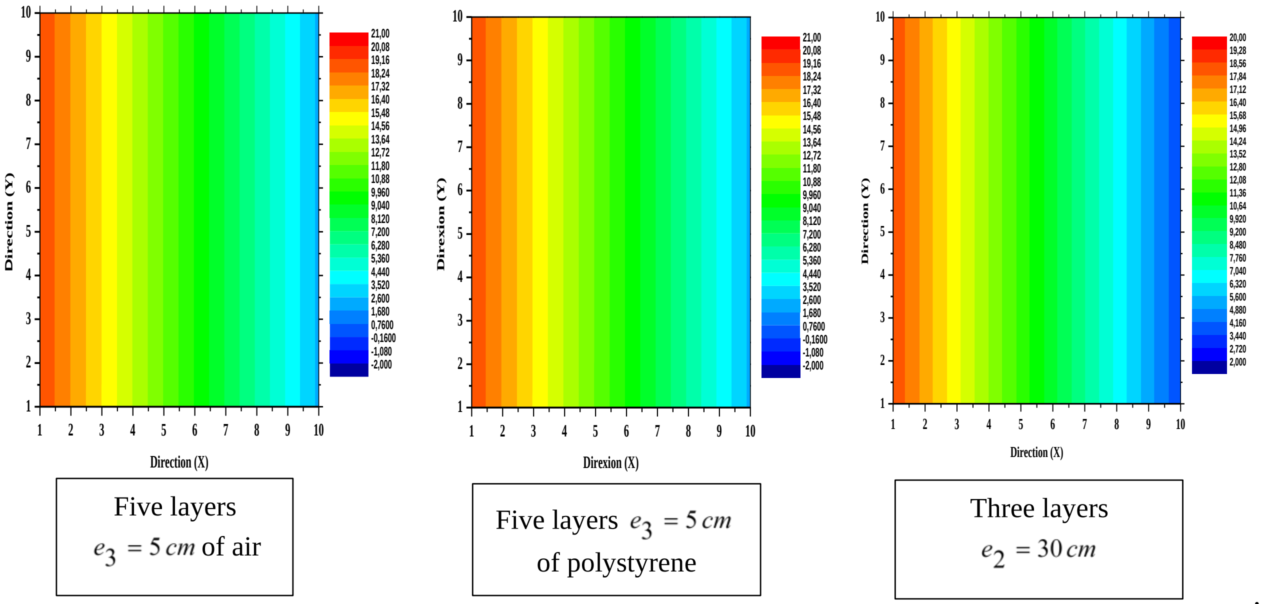

Fig 7. 2D temperature fields through the multilayer wall in adiabatic case.

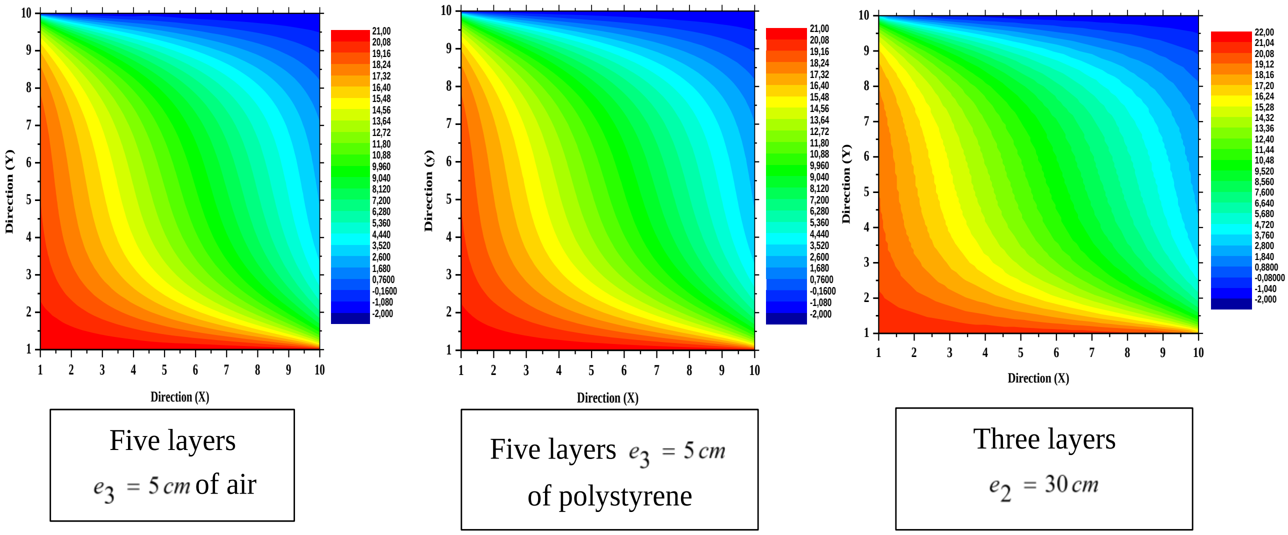

Fig 8. 2D temperature fields through the multilayer wall in the case 1.

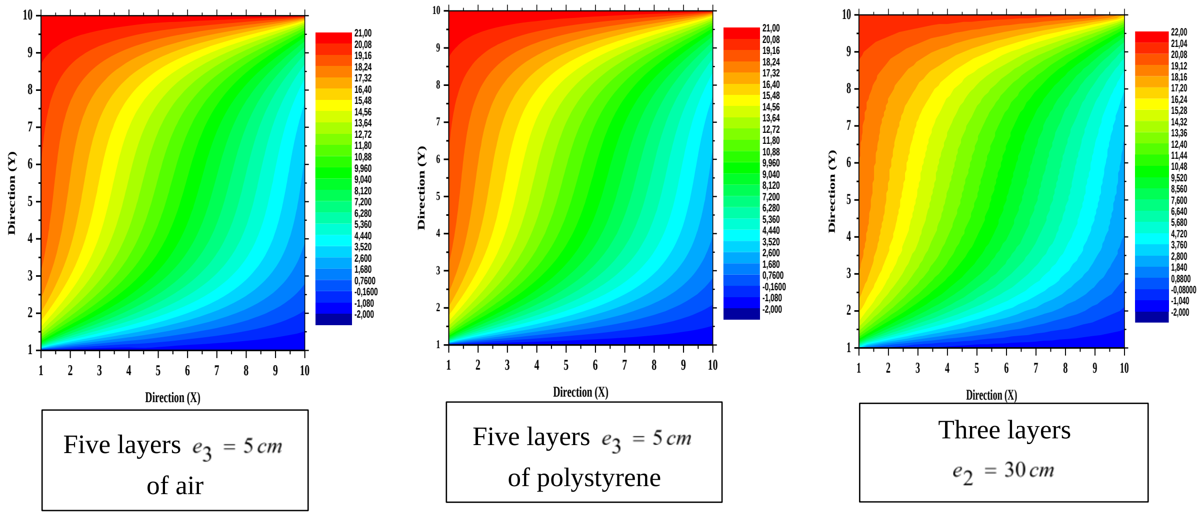

Fig 9. 2D temperature fields through the multilayer wall in the case 2.

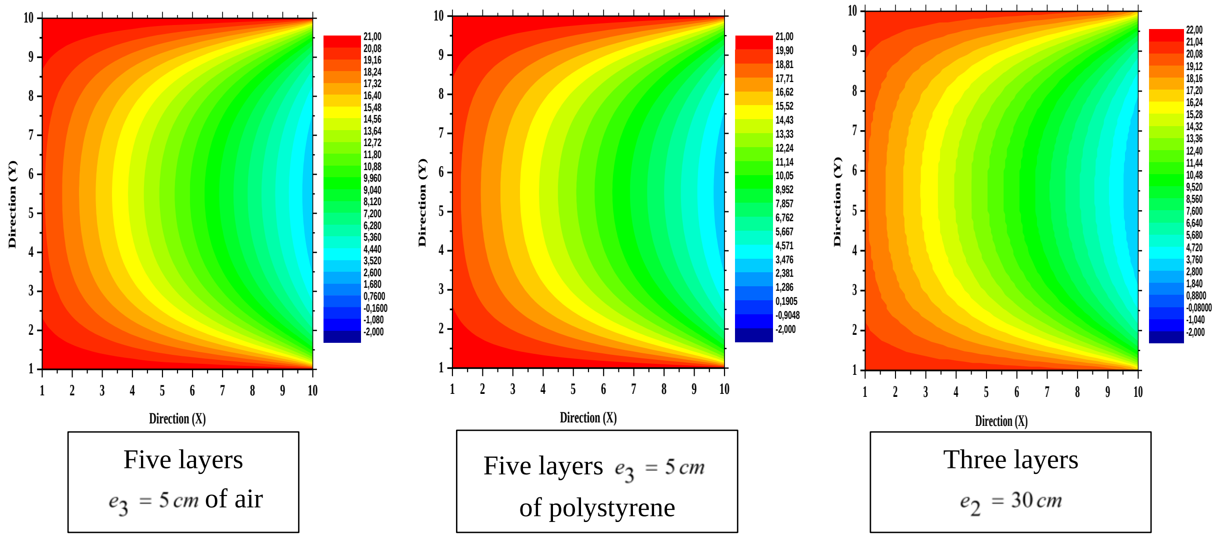

Fig 10. 2D temperature fields through the multilayer wall in the case 3.

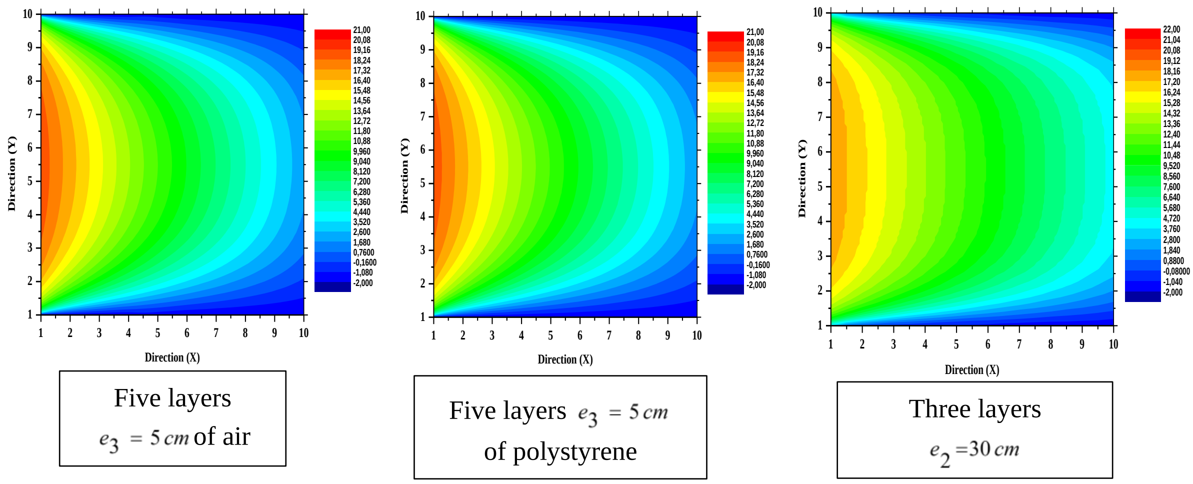

Fig 11. 2D temperature fields through the multilayer wall in the case 4.

Conclusion.

In the present work, the two-dimensional transient heat equation through a multilayers wall was resolved using ADI method, a Fortran program was carried out to study heat transfer with a convective exchange on the left and right sides (the values of the exchange coefficients, as well as the values of internal and external temperatures, are those of the city of Batna according to the Algerian technical regulatory document (DTR). For the top and bottom sides, different conditions are used, after validation of the program, it was used to investigate the thermal insulation in the building sector by taking the most common exterior walls configurations in Algeria.

The results showed that for a 15 cm or a 20 cm wall made of brick with 2 cm thickness of plaster inside and 2 cm mortar outside, give bad thermal insulation, in the other hand it was observed that the conditions imposed on the top and bottom of the wall have practically no influence on the internal temperature of the multilayers, while these are the both most used configurations in individual constructions. The temperature of a point in the middle of the internal plaster face was 18.1 and 18.5 ° C respectively for the first and the second configuration, while the internal temperature air is 21 ° C, this means that this temperature difference will be offset by excessive use of energy for heating.

In the case of thicker walls, and by comparing the three-layer wall made of 30 cm brick and a five-layer wall with the intermediate layer composed either of an air gap or polystyrene of 5 cm thickness, it noted that the temperatures of the internal face are around 19.25 ° C ± 0.5 ° C for the three configurations, but knowing that the lower price is obtained with the multilayer wall with an air gap, the choice will be made for this last configuration which ensures good thermal insulation with the lowest cost.

In the other hand, it was found that there is practically no influence of the top and bottom boundary conditions on the three layers wall (15 and 20 cm), unlike, in the case of three-layer wall made of 30 cm of brick, or the case of five-layer wall with air gap or polystyrene, it was noted that the internal temperatures are sensitive to the top and bottom boundary conditions.

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NOMENCLATURE

c_p Specific heat (J. Kg^{-1}.K^{-1})

h Heat transfer coefficient (W.m^{-2}.K^{-1})

e Layer thickness (m)

m Number of nodes following y

n Number of layers

L Wall hight

t Time (s)

T Temperature (°C)

T_{f_1} Indoor temperature (°C)

T_{f_2} Outdoor temperature (°C)

Greek symbols

λ Thermal conductivity ($W.m^{-1}.K^{-1}$)

ρ Mass density ($kg .m^{-3}$)

α Thermal diffusivity ($m^2.s^{-1}$)

Subscripts

B Bottom

f Final

T Top

1,..5 Number of layer