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Comparative study of two models to estimate solar radiation on an inclined surface

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Abstract - *Our work is focused on the study of the solar radiation received on three inclined surface: 32°, 60° and 90° facing south. So, we proposed two theoretical models: Brichambaut and Liu Jordan models. Brichambaut model takes into account two input parameters (local time, day number), Liu & Jordan takes into account the measures of the solar irradiance (diffuse and global radiations received on horizontal plane). We tested a day 10-02-2011. We noted that the Brichambaut model gives best results, the maximum relative errors are equal to: -0.43 %, -0.60 %, 0.76 % respectively for the angles 32°, 60°, 90°. While for the Liu & Jordan model we obtained the following errors: 1.55 %, 1.56 %, and 3.92 %.*

Résumé – *Notre travail est focalisé sur l'étude du rayonnement solaire reçu sur trois surfaces inclinées: 32°, 60° et 90° orientées vers le sud. Donc, nous avons proposé deux modèles théoriques: les modèles de Brichambaut et de Liu & Jordan. Le modèle de Brichambaut prend en compte deux paramètres d'entrée (l'heure locale et le numéro du jour), le modèle de Liu & Jordan prend en compte les mesures de l'éclairement solaire (rayonnements diffus et global reçu sur un plan horizontal). Nous avons testé la journée 02.10.2011. Nous avons constaté que le modèle de Brichambaut donne de meilleurs résultats, les erreurs maximales relatives sont égales à: -0.43 %, -0.60 %, 0.76 % respectivement pour les angles 32°, 60°, 90°. Alors que pour le modèle de Liu & Jordan, nous avons obtenu les erreurs suivantes: 1.55 %, 1.56 % et 3.92 %.*

Keywords: Rayonnement solaire - Station radiométrique - Modèles empirique.

1. INTRODUCTION

The radiometric station was installed on the roof of Solar Radiation Laboratory of URAER (Latitude: $\phi = 32.36°$N, Longitude: $\phi = 3.81°$E, altitude: $z = 460$ m, albedo: $\rho = 0.3$) has two parts: a fixed one which consists of four pyranometers to measure the total radiation on horizontal and inclined surface (32°, 60°, and 90°) facing south.

A moving part, which is able to track the sun from sunrise to sunset, which is pointed at the sun disk for measuring the direct radiation (beam), and an pyranometer , for the measurement of diffuse radiation on horizontal surface (Fig. 1).

All systems described above are related to Campbell Scientific CR10X data acquisition. It is connected by RS232, to a computer where we installed software for reading data.

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Fig. 1: Radiometric devices (Sun tracker)

① Ball sunshade: to eliminate the beam radiation

② Pyranometer Eko: to measure the diffuse radiation on horizontal plane

③ Pyranometer Eko: to measure the direct radiation

④ Two Pyranometer Eppley: to measure the global radiation on tilted plane: 60°, 90°

⑤ Pyranometer Eko: to measure the global radiation on horizontal

⑥ Pyranometer Eppley: to measure the global radiation on 32°

With these experimental devices we were able to build and exploit a data base of six years, we have studied modelling, and we developed models that allow estimation of the components of solar radiation (diffuse, direct and global).This study is devoted to validate two models (Brichambaut and Liu & Jordan), which are valid for clear skies.

However, these models are generally verified for a cloudless sky. Liu & Jordan model takes into account not only the clarity of the sky but also the isotropy of the sky, Brichambaut model takes into account the clarity of the sky, but contains only one input parameter (the height of the sun).

The angles choosing in this work are 32°, 60° and 90° and which correspond respectively to the optimum angle for the year, the optimum angle for the winter and the vertical plane facing south that is used in the bioclimatic.

2. MATHEMATICAL MODELS

2.1 Geometric and atmospheric formulas

2.1.1 Hour angle

The hour angle ω is the angular displacement of the local meridian of the east from the west due to rotation of the earth on its axis at 15° per hour, can be defined by:

$$\omega = 15 \times (t_{sv} - 12) \tag{1}$$

$$t_{sv} = t_l - 1 + \left(E_t / 60\right) + \lambda / 15 \tag{2}$$

E_t , t_l are respectively the equation of time and local time [2].

2.1.2 Declination angle

It is the angle between earth-sun line and the equatorial plane, [3]:

$$\delta = 23.45 \times \sin\left(\frac{(284 + j)}{265} \times 360\right) \tag{3}$$

2.1.3 Height of the sun

It's the angle between the distance sun-earth and horizontal plane of site, represented by [2]:

$$\sin(h) = \cos(\varphi) \times \cos(\delta) \times \cos(\omega) + \sin(\varphi) \times \sin(\delta) \tag{4}$$

2.1.4 Incidence angle

It's angle between the beam radiation on a surface and the normal to that surface, given by:

$$\cos(i) = \sin(\delta) \times \sin(\varphi - \beta) + \cos(\delta) \times \cos(\varphi - \beta) \times \cos(\omega) \tag{5}$$

where, β tilted angle.

2.1.5 Linke turbidity factor

The Linke's turbidity factor T_L is defined as the number of dry atmospheres that would produce the same total attenuation of direct solar radiation as that produced by the real atmosphere (presence of water vapor and aerosol), is given by the following expression, [1]:

$$T_L = T_1 + T_2 + T_3 \tag{6}$$

$$T_1 = (2.4 - 0.9 \times \sin(\varphi)) + 0.1 \times (2 + \sin(\varphi)) \times A - 0.2 \times z - (1.22 + 0.14 \times A) \times (1 - \sin(h)) \tag{7}$$

$$T_2 = (0.89)^z \tag{8}$$

$$T_3 = (0.9 + 0.4 \times A) \times (0.63)^z \tag{9}$$

$$A = \sin\left(\frac{360}{365} \times (j - 121)\right) \tag{10}$$

2.2 Empirical models

2.2.1 Brichambautt model - Direct solar radiation

The direct solar radiation estimated on inclined surface without being diffused by the atmosphere:

$$I_\beta = I_0 \times c(j) \times \sin(h) \exp(-T_L \times m \times k_0) \quad (11)$$

Where, k_0 is the attenuation coefficient for a pure an dry atmosphere, given by Kasten [1], as follows:

$$1/k_0 = 0.9 + (9.4 / m) \quad (12)$$

$c(j)$, m and I_0 are are respectively the Correction factor of the earth's orbit, air mass and extraterrestrial irradiance at normal incidence, [1, 6].

$$m = 1 / \sin(h) \quad (13)$$

To find the direct radiation on an inclined plane, we multiplied expression (11) by the formula (5).

2.2.2 Brichambaut model - Diffuse solar radiation

Many researchers have proposed the empirical or semi empirical models who calculate the diffuse radiation on a collector for any inclination, the Brichambaut model, takes into account following parameters (**Table 1**):

- the scattering by the hemispherical component
- the scattering by the circumsolar component
- the scattering by the horizon brightening components
- the ground reflexion (Albedo=0.3 for Ghardaïa region).

All these components will contribute to the attenuation of solar radiation and are estimated by the formula:

$$d_\beta = d_{iso} + d_{cs} + d_{hz} + d_{ref} \quad (14)$$

All terms of this equation are stored in the **Table 1** below.

2.2.3 Liu & Jordan model - Direct solar radiation

The direct solar radiation estimated on inclined surface given by:

$$I_\beta = I_h \times \frac{\cos(i)}{\sin(i)} \quad (15)$$

where, I_h it's the direct irradiance measured on horizontal surface.

2.2.4 Liu & Jordan model - Diffuse solar radiation

We can to use the Liu & Jordan model which takes into account the isotropic part of the sky (hemispherical). In this case, the equation is: [2, 4]

$$d_\beta = d_h \times \left(\frac{1 + \sin(90° - \beta)}{2} \right) + \rho \times g_h \times \frac{1 - \cos(\beta)}{2} \quad (16)$$

where, d_h and g_h are respectively the diffuse and global irradiance measured on horizontal surface.

2.3 Total solar radiation

The total solar radiation measured or estimated on an inclined surface is given by:

$$g_\beta = I_\beta + d_\beta \tag{17}$$

Table 1: The different contribution of diffuse solar radiation

Type of contribution	Equations (1)	Equations (2)
The hemispherical Diffusion	$d_{iso} = (\delta_i + \delta_{ret}) \times \left(\frac{1+\sin(90-\beta)}{2}\right)$	$\delta_i = d_h - \delta_d \times \sin(h)$ where $d_h = I_0 \times \sin h \times \exp\left(-1+1.06\times\log(\sin h) + a_1 - \sqrt{a_1^2+b_1^2}\right)$ $\delta_h = I_0 \times \exp\left(-2.48+(\sin h) + a_2 - \sqrt{a_2^2+4\times b_2^2}\right)$ $\delta_{ret} = 0.9\times(\rho-0.2)\times (d_k + I_n \times \sin h)\times \exp\left(-\frac{4}{\sqrt{T_2+T_3}}\right)$ $a_1 = 1.1$ $a_2 = 3.1 - (0.4 \times b_2)$ $b_1 = \log(T_2+T_3) - 2.8 + 1.02\times(1-\sin h)^2$ $b_2 = \log(T_2+T_3) - 2.28 - 0.5\times\log(\sin h)$
The circumsolar Component	$d_{cs} = \delta_d \times \cos i$	
The horizon brightening components	$d_{hz} = \delta_{hz} \times \sin(h) \times \cos(90-\beta)$	$\delta_{hz} = -\frac{0.02\times a_3}{a_3^2 + a_3\times b_3 + 1.8}$ where $a_3 = \log(T_2+T_3) - 3.1 - \log(\sin h)$ $b_3 = \exp(0.2 + 1.75\times\log(\sin h))$
The ground reflexion components	$d_{ref} = (\delta_d + I_n \times \sin h)\times\rho\left(\frac{1+\sin(90-\beta)}{2}\right)$	

2.4 Daily global irradiation

The numerical integrals which we adopted in this work to calculate the daily radiation are given by:

- The Daily global irradiation on inclined surface obtained by the measurement of the global irradiance

$$G_\beta = \int_{t_l}^{t_c} g_\beta \times dt \tag{18}$$

- The Daily global irradiation on inclined surface estimated by the Brichambaut model

$$G_{br} = \int_{t_l}^{t_c} g_{br} \times dt \tag{19}$$

- The Daily global irradiation on inclined surface estimated by the Liu & Jordan model

$$G_{lj} = \int_{t_l}^{t_c} g_{lj} \times dt \tag{20}$$

2.5 Statistical formulas

The relative errors between the measured and estimated illumination by the two models are given as follows:

- Relative error between the maximum of the global irradiance measured and that obtained by the Brichambaut model

$$e_{br} = \frac{\max(g_\beta) - \max(g_{br})}{\max(g_\beta)} \tag{21}$$

- Relative error between the maximum of the global irradiance measured and that obtained by Liu & Jordan model

$$e_{lj} = \frac{\max(g_\beta) - \max(g_{lj})}{\max(g_\beta)} \tag{22}$$

- Relative error between the maximum of the global irradiation measured and that obtained by the Brichambaut model

$$E_{br} = \frac{\max(G_\beta) - \max(G_{br})}{\max(G_\beta)} \tag{23}$$

- Relative error between the maximum of the global irradiation measured and that obtained by the Liu & Jordan model

$$E_{lj} = \frac{\max(G_\beta) - \max(G_{lj})}{\max(G_\beta)} \tag{24}$$

3. RESULTS AND DISCUSSION

To validate the models presented in this work, we estimated three components of solar radiation, which are diffuse, direct and global irradiance received on three inclined planes.

We used the results of measurements on a horizontal plane for computed for three components of solar radiation (diffuse, direct and global) on inclined plane. Then, we compared the results with measurements obtained by a statistical study and graphics (**Table 1** and figures).

Figure 2 shows the validation study of two models of global solar irradiance and the measurements of global solar irradiance on an inclined plane at 32°. So, we reconstructed two groups of graphics, who gives respectively the measurements values as a function of each model for an inclination 32° (Fig. 2-a-, Fig. 2-c-), and the measurements values as a function of time (Fig. 2-b-, Fig. 2-d-).

Figure 2-a- and figure 2-c- shows two linear curves obtained from the principle of least squares. We found that the two curves have linear form $y = a \times x$ where a is nearly equal to 1.

Figure 2-b- and figure 2-d- shows the evolution of solar irradiance measured and estimated by each model versus time. We found that the two curves are confounded, where the errors are very low especially for the Brichambaut model.

The same results were found for the other figures (Fig. 3 and Fig. 4, meaning that for the angles 60° and 90° except for the vertical tilt ($\beta = 90°$), where there a small difference between the measures and estimates.

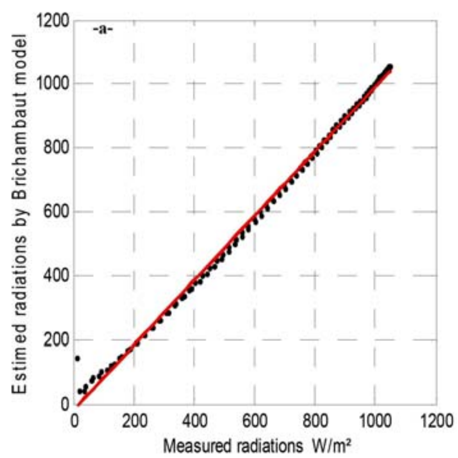
Therefore, the solar radiation obtained by the two models is in good conformity with the measures especially the Brichambaut model.

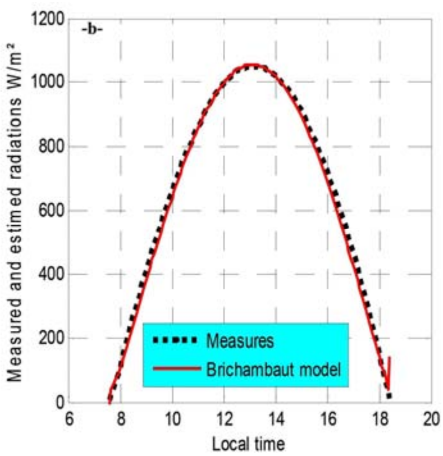
The quantitative (statistical) was conducted to evaluate the difference between the measured values with those estimated of the total solar irradiance and total solar irradiation by the two models and for the three inclinations.

Table 2, shows the relative errors (equations {equ. (21)}, {equ. (22)}, {equ. (23)} and {equ. (24)}) between the solar irradiance measured and estimated by two models, and the relative error between the daily global irradiation measured and estimated by Brichambaut and Liu & Jordan models.

We see that the largest errors are equal to 3.92 % and 4.43 % respectively for global irradiance and daily global irradiation for Liu & Jordan model and for vertical tilt (90°) facing south.

Against, the lowest values are equal to 0.43 % and 0.5 % respectively for illumination and daily global irradiation, correspond to the Brichambaut model. We also note that the minimum value of daily irradiation is 6664 Wh/m^2, which is considered an important value.





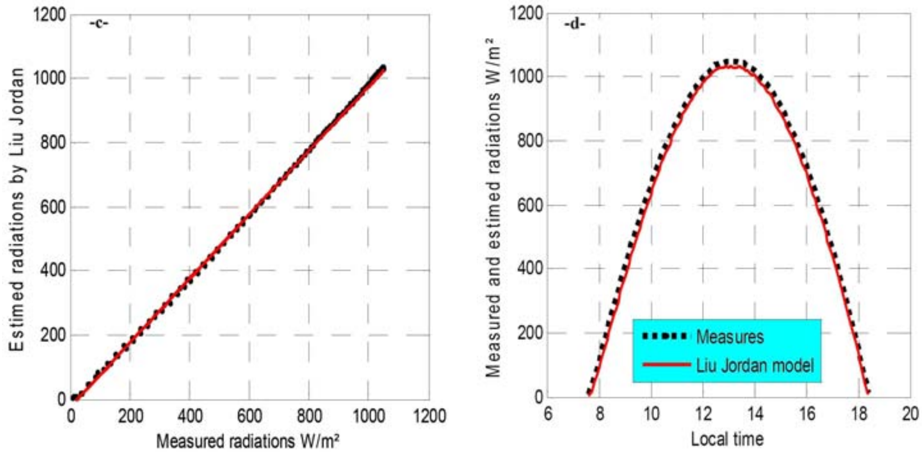


Fig. 2: Comparison between measured and computed global radiation at 32° using two models using two models

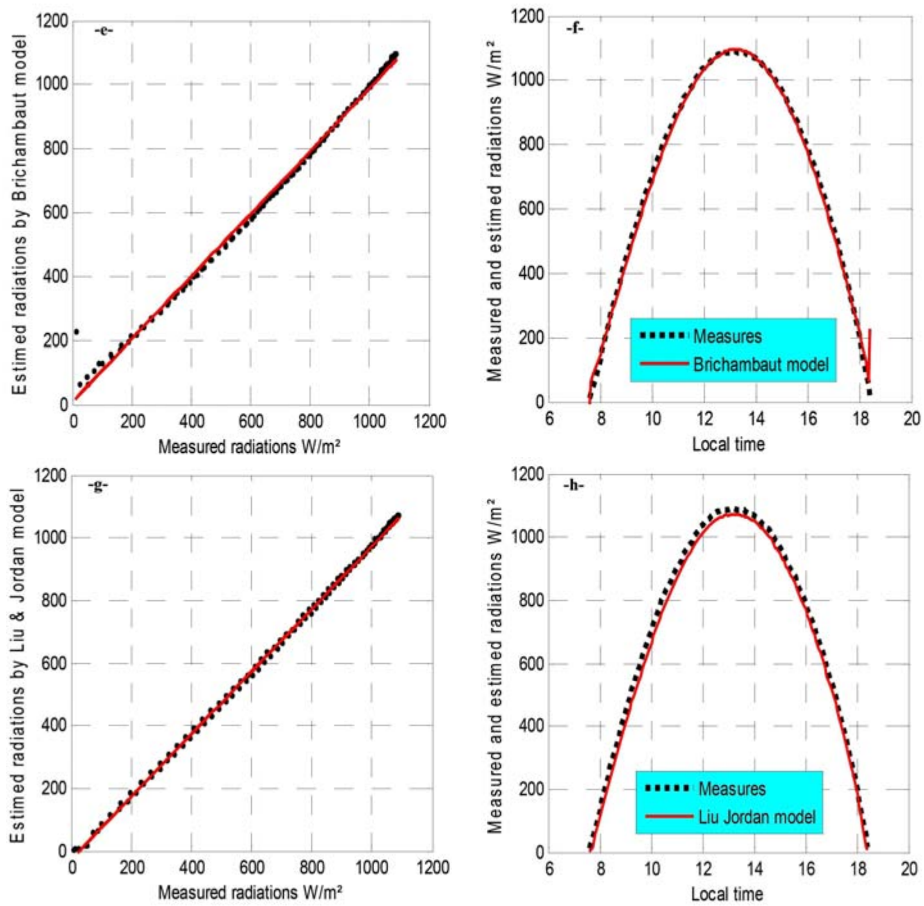


Fig. 3: Comparison between measured and computed global radiation at 60° using two models using two models

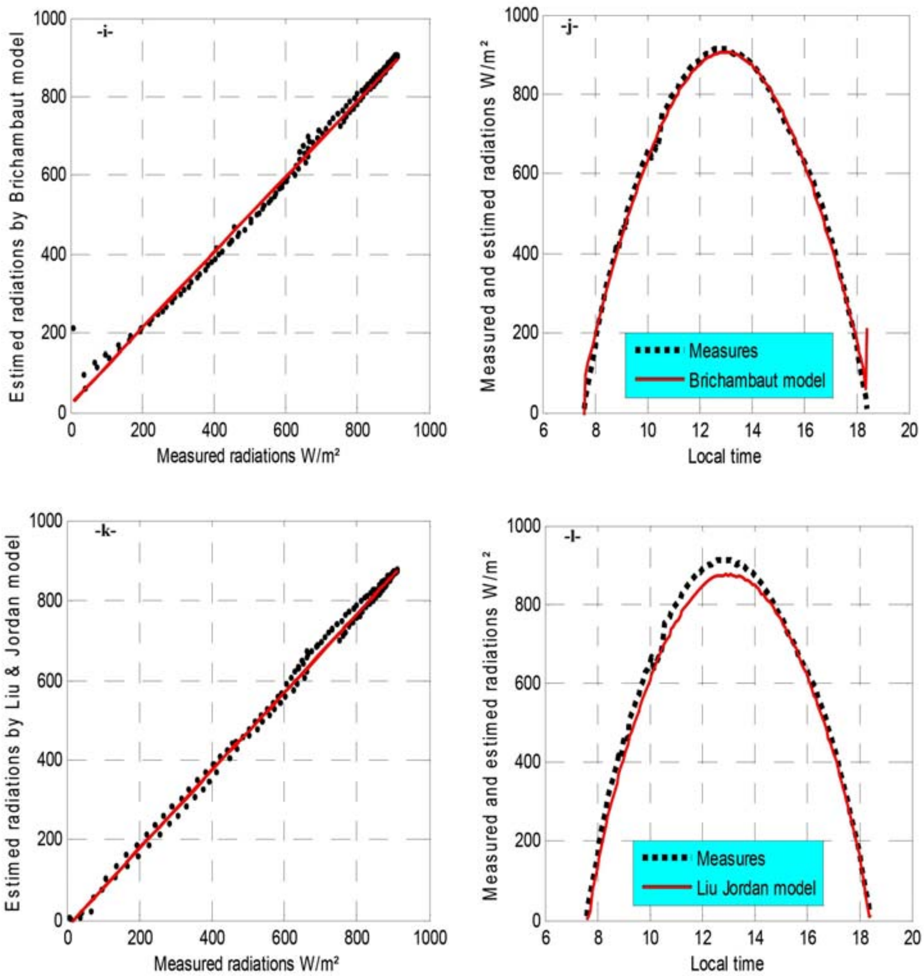


Fig. 4: Comparison between measured and computed
global radiation at 90° using two models using two models

Table 2: Relative error between the solar radiation measured and estimated

	e_{br} (%)	e_{lj} (%)	G_β (Wh/m^2)	G_{br} (Wh/m^2)	G_{lj} (Wh/m^2)	E_{br} (%)	E_{lj} (%)
32°	-0.43	1.55	7336	7185	7090	2.60	3.35
60°	-0.60	1.56	7842	7788	7563	0.70	3.55
90°	0.76	3.92	6697	6664	6400	0.50	4.43

4. CONCLUSION

This work has enabled us to conclude that the proposed models are in good agreements with measured data and can be an appropriate simulation for global radiation incident on an inclined plane and facing south, however the Brichambaut model is more interesting because the relative errors compared with measures are very small.

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